# 1.2. Prüfungsaufgaben zur Bruchrechnung

**Aufgabe 1: Addition und Subtraktion**

Fasse zusammen und vereinfache soweit möglich:

a) $\frac{x + ny}{n} - \frac{x - ny}{n}$

b) $\frac{x - 7}{4x} - \frac{3x + 4}{4x} - \frac{8x - 5}{4x}$

c) $\frac{x}{x - y} - \frac{y}{x - y}$

d) $\frac{x + y}{2xy} + \frac{x + z}{2xz} + \frac{y + z}{2yz}$

e) $\frac{x^2}{x + 1} - x$

f) $\frac{2}{3a} + \frac{1}{b}$

g) $\frac{5}{3x} - \frac{11}{9} - \frac{4}{x^2}$

h) $\frac{3}{9a} + \frac{1}{6a^2} + \frac{1}{4ab}$

i) $\frac{a}{a - b} - 1$

j) $1 - \frac{2a}{a + b}$

k) $\frac{7a - 4}{4a + 4} - \frac{a - 2}{2a + 2}$

l) $\frac{4b - 2}{2b + 4} - \frac{8b - 7}{6b + 12} - \frac{2b - 5}{10b + 20}$

**Lösungen**

a) $\frac{x+ny}{n} - \frac{x-ny}{n} = 2y$ (2)

b) $\frac{x-7}{4x} - \frac{3x+4}{4x} - \frac{8x-5}{4x} = -\frac{10x+6}{4x} = -\frac{5x+3}{2x}$ (2)

c) $\frac{x}{x-y} - \frac{y}{x-y} = 1$ (2)

d) $\frac{x+y}{2xy} + \frac{x+z}{2xz} + \frac{y+z}{2yz} = \frac{xy+xz+yz}{xyz}$ (2)

e) $\frac{x^2}{x+1} - x = -\frac{x}{x+1}$ (2)

f) $\frac{2}{3a}+\frac{1}{b} = \frac{2b+3a}{3ab}$ (1)

g) $\frac{5}{3x}-\frac{11}{9}-\frac{4}{x^2} = \frac{15x}{9x^2}-\frac{11x^2}{9x^2}-\frac{36}{9x^2} = \frac{15x-11x^2-36}{9x^2}$ (2)

h) $\frac{3}{9a}+\frac{1}{6a^2}+\frac{1}{4ab} = \frac{4ab}{12a^2b} + \frac{2b}{12a^2b} + \frac{3a}{12a^2b} = \frac{4ab+2b+3a}{12a^2b}$ (2)

i) $\frac{a}{a-b} - 1 = \frac{a}{a-b} - \frac{a-b}{a-b} = \frac{a-a+b}{a-b} = \frac{b}{a-b}$ (3)

j) $1-\frac{2a}{a+b} = \frac{a+b-2a}{a+b} = \frac{-a+b}{a+b}$ (2)

k) $\frac{7a-4}{4a+4} - \frac{a-2}{2a+2} = \frac{7a-4}{4(a+1)} - \frac{a-2}{2(a+1)} = \frac{7a-4-2(a-2)}{4(a+1)} = \frac{5a}{4a+4}$ (3)

l) $\frac{4b-2}{2b+4} - \frac{8b-7}{6b+12} - \frac{2b-5}{10b+20} = \frac{60b-30}{30(b+2)} - \frac{40b-35}{30(b+2)} - \frac{6b-15}{30(b+2)} = \frac{14b+20}{30(b+2)} = \frac{7b+10}{15b+30}$ (3)

**Aufgabe 2: Multiplikation und Division**

Berechne die folgenden Ausdrücke und vereinfache anschließend soweit möglich:

a) $8ab : \frac{4a}{5b}$

b) $(7x-4) : \frac{14x-8}{y^2}$

c) $\left(\frac{b}{a}-1\right):\left(\frac{a-b}{ab}\right)$

d) $\frac{3a^2+6ab}{6xy-3y^2} : \frac{4ab+8b^2}{8x^2-4xy}$

**Lösungen**

a) $8ab : \frac{4a}{5b} = \frac{8ab\cdot 5b}{4a} = 10b^2$ (2)

b) $(7x-4) : \frac{14x-8}{y^2} = \frac{(7x-4)y^2}{2(7x-4)} = \frac{y^2}{2}$ (2)

c) $\left(\frac{b}{a}-1\right):\left(\frac{a-b}{ab}\right) = \left(\frac{b-a}{a}\right)\cdot\left(\frac{ab}{a-b}\right) = \frac{-ab(a-b)}{a(a-b)} = -b$ (3)

d) $\frac{3a^2+6ab}{6xy-3y^2} : \frac{4ab+8b^2}{8x^2-4xy} = \frac{3a(a+2b)}{3y(2x-y)} \cdot \frac{4x(2x-y)}{4b(y+2b)} = \frac{ax}{by}$ (3)

**Aufgabe 3: Vereinfachen von Summen und Differenzen mit Hilfe der binomischen Formeln**
Berechne die folgenden Ausdrücke und vereinfache anschließend soweit möglich:

a) $\frac{1}{x+y}+\frac{1}{x-y}$

b) $\frac{u+1}{u+2}-\frac{u-2}{u-1}$

c) $\frac{a+b}{a-b}-\frac{2ab-2b^2}{a^2-2ab+b^2}$

d) $\frac{a}{a-b}-\frac{3ab-b^2}{a^2-b^2}$

e) $\frac{a+4}{a-4}-\frac{a-4}{a+4}-\frac{64}{a^2-16}$

f) $\frac{3a+b}{a+b}-\frac{a-3b}{a-b}-2$

g) $\frac{3a+5b}{a+b}-\frac{a-3b}{a-b}-2$

h) $\frac{a}{a-b}-\frac{b}{b-a}-\frac{2ab}{a^2-b^2}$

i) $\frac{x+5}{x^2-2x-3}+\frac{9x-7}{x^2-x-6}-\frac{4x+3}{x^2+3x+2}$

j) $\frac{1}{x-1}-\frac{4}{4x+4}+\frac{x-2}{x^2-x-2}-\frac{3x+6}{3x^2+3x+3}$

k) $\frac{2x-y}{2x-2y}-\frac{x-y}{3x+3y}+\frac{y(18y+2x)}{12x^2-12y^2}$

**Lösungen**

a) $\frac{1}{x+y}+\frac{1}{x-y} = \frac{x-y+x+y}{(x+y)(x-y)} = \frac{2x}{x^2-y^2}$ (2)

b) $\frac{u+1}{u+2} - \frac{u-2}{u-1} = \frac{(u+1)(u-1)-(u-2)(u+2)}{(u+2)(u-1)} = \frac{3}{u^2+u-2}$ (3)

c) $\frac{a+b}{a-b}-\frac{2ab-2b^2}{a^2-2ab+b^2} = \frac{(a+b)(a-b)}{(a-b)^2}-\frac{2ab-2b^2}{(a-b)^2} = \frac{a^2-b^2}{(a-b)^2}-\frac{2ab-2b^2}{(a-b)^2} = \frac{a^2-2ab+b^2}{(a-b)^2} = 1$ (4)

d) $\frac{a}{a-b}-\frac{3ab-b^2}{a^2-b^2} = \frac{a(a+b)}{(a-b)(a+b)}-\frac{3ab-b^2}{(a-b)(a+b)} = \frac{a^2+ab-3ab+b^2}{(a-b)(a+b)} = \frac{(a-b)^2}{(a-b)(a+b)} = \frac{a-b}{a+b}$ (3)

e) $\frac{a+4}{a-4} - \frac{a-4}{a+4} - \frac{64}{a^2-16} = \frac{(a+4)^2-(a-4)^2-64}{(a-4)(a+4)} = \frac{16a-64}{a^2-16} = \frac{16(a-4)}{(a-4)(a+4)} = \frac{16}{a+4}$ (5)

f) $\frac{3a+b}{a+b} - \frac{a-3b}{a-b} - 2 = \frac{(3a+b)(a-b)-(a-3b)(a+b)-2(a+b)(a-b)}{(a+b)(a-b)} = 0$ (4)

g) $\frac{3a+5b}{a+b} - \frac{a-3b}{a-b} - 2 = \frac{(3a+5b)(a-b)-(a-3b)(a+b)-2(a+b)(a-b)}{(a+b)(a-b)} = \frac{4ab}{a^2-b^2}$ (4)

h) $\frac{a}{a-b} - \frac{b}{b-a} - \frac{2ab}{a^2-b^2} = \frac{a^2+b^2}{a^2-b^2}$ (3)

i) $\frac{x+5}{x^2-2x-3} + \frac{9x-7}{x^2-x-6} - \frac{4x+3}{x^2+3x+2} = \frac{6}{x-3}$ (4)

j) $\frac{1}{x-1} - \frac{4}{4x+4} + \frac{x-2}{x^2-x-2} - \frac{3x+6}{3x^2+3x+3} \quad \frac{3}{(x-1)\left(x^2+x+1\right)} = \frac{3}{x^3-1}$ (4)

k) $\frac{2x-y}{2x-2y} - \frac{x-y}{3x+3y} + \frac{y(18y+2x)}{12x^2-12y^2} = \frac{(2x-y)(3x+3y)-2(x-y)^2+y(9y+x)}{6(x-y)(x+y)} = \frac{4x^2+8xy+4y^2}{6(x-y)(x+y)} = \frac{4(x+y)^2}{6(x-y)(x+y)} =$
$\frac{2(x+y)}{3(x-y)}$ (6)

**Aufgabe 4: Vereinfachen von Produkten mit Hilfe der binomischen Formeln**
Fasse zusammen und vereinfache anschließend soweit möglich:

a) $\frac{p^2 - q^2}{p^2 + q^2} \cdot \frac{p + q}{p - q}$

b) $\frac{k - 1}{18k} \cdot \frac{12k^2}{1 - k}$

c) $\frac{v^2 + 4v + 4}{3t - 3} \cdot \frac{9 - 9t}{v^2 + 5v + 6}$

d) $(a - b)\left(\frac{1}{a} + \frac{1}{b}\right)$

e) $\left(\frac{x}{y} - \frac{y}{x}\right)\left(\frac{y}{x} + \frac{x}{y}\right)$

f) $\frac{1}{a - 1} \cdot \frac{a^2 - 1}{1 - a^2} \cdot \left(\frac{1 - a}{1 + a}\right)^2 \cdot \frac{a^3 - a^2 - a + 1}{a^2 - 2a + 1}$

g) $\left(\frac{a}{b} - \frac{b}{a}\right)^2$

h) $\left(\frac{m}{2} + \frac{3n}{4}\right) \cdot \left(\frac{m}{2} - \frac{3n}{4}\right)$

i) $\frac{x^2y - 2x^2}{8uv} \cdot \frac{4u^2v}{3xy^2 - 12x}$

j) $\frac{4a^2 + 12ab + 9b^2}{15x^2y} \cdot \frac{55x^2}{18a + 27b}$

k) $\frac{5mn - 7n^2}{9a^2 + 12ab + 4b^2} \cdot \frac{12a + 8b}{25m^2 - 35mn}$

l) $\frac{ab^2}{2c^2d^2 - 18c^2} \cdot \frac{6c^2d + 18c^2}{a^2b}$

**Lösungen**

a) $\dfrac{(p+q)^2}{p^2+q^2}$

b) $-\dfrac{2k}{3}$

c) $-\dfrac{3(v+2)}{v+3}$

d) $\dfrac{a^2-b^2}{ab}$

e) $\dfrac{x^4-y^4}{x^2y^2}$

f) $\dfrac{1}{a-1}\cdot\dfrac{a^2-1}{1-a^2}\cdot\left(\dfrac{1-a}{1+a}\right)^2\cdot\dfrac{a^3-a^2-a+1}{a^2-2a+1}$

$$=\frac{1}{\cancel{(a-1)}}\cdot\frac{\cancel{(a-1)}\,\cancel{(a+1)}}{(1-a)\,\cancel{(1+a)}}\cdot\frac{\cancel{(1-a)^2}}{(1+a)^2}\cdot\frac{a^2(a-1)-(a-1)}{\cancel{(a-1)^2}}$$

$$=\frac{(a-1)(a^2-1)}{(1-a)(1+a)^2}=\frac{(-1)\cancel{(1-a)}(a-1)\cancel{(a+1)}}{\cancel{(1-a)}(1+a)^{\cancel{2}}}=\mathbf{\frac{1-a}{a+1}}$$

g) $\left(\dfrac{a}{b}-\dfrac{b}{a}\right)^2=\left(\dfrac{a}{b}\right)^2-2\cdot\dfrac{a}{b}\cdot\dfrac{b}{a}+\left(\dfrac{b}{a}\right)^2=\dfrac{a^2}{b^2}-2+\dfrac{b^2}{a^2}=\dfrac{a^2a^2-2a^2b^2+b^2b^2}{a^2b^2}=\dfrac{(a^2-b^2)^2}{a^2b^2}$ (4)

h) $\left(\dfrac{m}{2}+\dfrac{3n}{4}\right)\cdot\left(\dfrac{m}{2}-\dfrac{3n}{4}\right)=\dfrac{2m+3n}{4}\cdot\dfrac{2m-3n}{4}=\dfrac{4m^2-9n^2}{16}$ (3)

i) $\dfrac{x^2y-2x^2}{8uv}\cdot\dfrac{4u^2v}{3xy^2-12x}=\dfrac{4x^2u^2v(y-2)}{24xuv(y^2-4)}=\dfrac{xu(y-2)}{6(y-2)(y+2)}=\dfrac{xu}{6(y+2)}$ (3)

j) $\dfrac{4a^2+12ab+9b^2}{15x^2y}\cdot\dfrac{55x^2}{18a+27b}=\dfrac{(2a+3b)^2}{15x^2y}\cdot\dfrac{55x^2}{9(2a+3b)}=\dfrac{11(2a+3b)}{27y}$ (4)

k) $\dfrac{5mn-7n^2}{9a^2+12ab+4b^2}\cdot\dfrac{12a+8b}{25m^2-35mn}=\dfrac{n(5m-7n)}{(3a+2b)^2}\cdot\dfrac{4(3a+2b)}{5m(5m-7n)}=\dfrac{4n}{5m(3a+2b)}$ (4)

l) $\dfrac{ab^2}{2c^2d^2-18c^2}\cdot\dfrac{6c^2d+18c^2}{a^2b}=\dfrac{6ab^2c^2(d+3)}{2a^2bc^2(d^2-9)}=\dfrac{6ab^2c^2(d+3)}{2a^2bc^2(d-3)(d+3)}=\dfrac{3b}{a(d-3)}$ (4)

**Aufgabe 5: Vereinfachen von Quotienten mit Hilfe der binomischen Formeln**
Fasse zusammen und vereinfache anschließend soweit möglich:

a) $\frac{a^3 + a^2 b}{c^2 + 1} : \frac{a^3 - ab^2}{c^2 - c}$

b) $\frac{r^4 - 1}{rs - s^2} : \frac{4r + 4}{r^2 - rs - r + s}$

c) $39g^2 h^2 : \frac{52g}{9h}$

d) $(7 - k) : \frac{k - 7}{-k - 7}$

e) $\left( u^2 + \frac{u}{v} \right) : \frac{u}{v}$

f) $\left( \frac{a}{b} - \frac{c}{d} \right) : \left( \frac{a}{b} + \frac{c}{d} \right)$

g) $\left( x - \frac{1}{x} \right) : \left( x + \frac{1}{x} \right)$

h) $\left( \frac{w}{2} - \frac{2}{w} \right) : (w + 2)$

i) $4y^2 z^3 \left( \frac{2x}{yz^2} - \frac{3x}{y^2 z} \right) : (3z - 2y)$

j) $\left( \frac{25a^2 - 9}{a^2 + 4a + 4} \cdot \frac{a^2 + 5a + 6}{b^3} \right) : \frac{5a - 3}{ab^3 + 2b^3}$

k) $\frac{x^2 + 2xy + y^2}{2x - 2y} : \frac{4x^2 - 4y^2}{2x^2 - 4xy + 2y^2}$

l) $\left( \frac{x}{y} - \frac{y}{x} \right) : \left( \frac{1}{y} - \frac{1}{x} \right)$

**Lösungen:**

a) $\frac{ac(c-1)}{(a-b)(c^2+1)}$

b) $\frac{(r-1)^2(r^2+1)}{4s}$

c) $\frac{27gh^3}{4}$

d) $k + 7$

e) $uv - 1$

f) $\frac{ad-bc}{ad+bc}$

g) $\frac{x^2-1}{x^2+1}$

h) $\frac{w-2}{2w}$

i) $-4xyz$

j) $\left(\frac{25a^2-9}{a^2+4a+4}\cdot\frac{a^2+5a+6}{b^3}\right):\frac{5a-3}{ab^3+2b^3} = \frac{(5a-3)(5a+3)(a+3)\cdot b^2\cdot(a+2)}{(a+2)\cdot b^2\cdot(5a-3)} = (5a+3)(a+3)$

k) $\frac{x^2+2xy+y^2}{2x-2y} : \frac{4x^2-4y^2}{2x^2-4xy+2y^2} = \frac{(x+y)^2}{2(x-y)}\cdot\frac{2(x-y)^2}{4(x^2-y^2)} = \frac{(x+y)^2}{2(x-y)}\cdot\frac{2(x-y)^2}{4(x-y)(x+y)} = \frac{x+y}{4}$ (4)

l) $\left(\frac{x}{y}-\frac{y}{x}\right):\left(\frac{1}{y}-\frac{1}{x}\right) = \left(\frac{x^2-y^2}{xy}\right):\left(\frac{x-y}{xy}\right) = \frac{(x-y)(x+y)}{xy}\cdot\frac{xy}{x-y} = x + y$ (4)

**Aufgabe 6: Vermischte Brüche**
Vereinfache soweit wie möglich:

a) $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}-\frac{y}{x}}$ (2)

b) $\dfrac{\frac{a}{b}\cdot\frac{c}{d}}{\frac{a}{b}-\frac{a+b}{d}}$ (2)

c) $\dfrac{\frac{n}{n^2-1}}{\frac{1}{n+1}-\frac{1}{n-1}}$ (2)

d) $\dfrac{s^2+t^2}{\frac{s}{s-t}-\frac{t}{s+t}}$ (3)

e) $\dfrac{\frac{2a}{a-3}-\frac{a}{a+4}}{\frac{a+11}{a^2+a-12}}$ (4)

f) $\dfrac{\frac{2a}{a-3}-\frac{a}{a+4}}{\frac{a+11}{a^2+a-12}}$ (4)

g) $\dfrac{\frac{4a^2-9b^2}{(2a+3b)^2}-\frac{2a+3b}{2a-3b}}{\frac{(2a+3b)^2}{4a^2-9b^2}-\frac{4a^2-9b^2}{4a^2+12ab+9b^2}}$ (5)

h) $\dfrac{1}{2+\frac{1}{3+\frac{1}{3}}}$ (5)

i) $\dfrac{x-y-\frac{x-y}{y+x}}{\frac{x}{y}-\frac{x}{x+y}}$ (5)

j) $\dfrac{\frac{a}{a+b}+\frac{b}{a-b}}{\frac{1}{a+b}-\frac{1}{b-a}}$ (5)

k) $\dfrac{x-\frac{1}{x}}{x-\frac{x}{x+\frac{1}{x}}}$ (5)

l) $\dfrac{a}{a+\frac{a}{1-\frac{a}{a-x}}}$ (5)

m) $$\frac{1}{k + \frac{1}{k + \frac{1}{k + \frac{1}{k}}}}$$ (5)

**Lösungen:**

a) $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}-\frac{y}{x}} = \dfrac{1}{x-y}$ (2)

b) $\dfrac{\frac{a}{b}\cdot\frac{c}{d}}{\frac{a}{b}-\frac{a+b}{d}} = \dfrac{ac}{ad-ab-b^2}$ (2)

c) $\dfrac{\frac{n}{n^2-1}}{\frac{1}{n+1}-\frac{1}{n-1}} = -\dfrac{n}{2}$ (2)

d) $\dfrac{s^2+t^2}{\frac{s}{s-t}-\frac{t}{s+t}} = s^2-t^2$ (3)

e) $\dfrac{\frac{2a}{a-3}-\frac{a}{a+4}}{\frac{a+11}{a^2+a-12}} = a$ (4)

f) $\dfrac{\frac{2a}{a-3}-\frac{a}{a+4}}{\frac{a+11}{a^2+a-12}} = -1$ (4)

g) $\dfrac{\frac{4a^2-9b^2}{(2a+3b)^2}-\frac{2a+3b}{2a-3b}}{\frac{(2a+3b)^2}{4a^2-9b^2}-\frac{4a^2-9b^2}{4a^2+12ab+9b^2}}$ (5)

h) $\dfrac{1}{2+\frac{1}{3+\frac{1}{3}}} = \dfrac{10}{23}$ (5)

i) $\dfrac{x-y-\frac{x-y}{y+x}}{\frac{x}{y}-\frac{x}{x+y}} = \dfrac{\frac{(x-y)(x+y)-(x-y)}{y+x}}{\frac{x(x+y)-xy}{x+y}} = \dfrac{\frac{(x-y)(x+y-1)}{y+x}}{\frac{x^2}{x+y}} = \dfrac{(x-y)(x+y-1)(y+x)}{(y+x)\cdot x^2}$

$= \dfrac{(x-y)(x+y-1)}{x^2}$ (5)

j) $\dfrac{\frac{a}{a+b}+\frac{b}{a-b}}{\frac{1}{a+b}-\frac{1}{b-a}} = \dfrac{\frac{a(a-b)+b(a+b)}{(a+b)(a-b)}}{\frac{b-a-(a+b)}{(a+b)(b-a)}} = \dfrac{\frac{a^2-ab+ab+b^2}{(a+b)(a-b)}}{\frac{-2a}{(a+b)(b-a)}} = \dfrac{\frac{a^2-ab+ab+b^2}{(a+b)(a-b)}}{\frac{2a}{(a+b)(a-b)}} =$

$\dfrac{(a^2+b^2)\cdot(a+b)(a-b)}{2a\cdot(a+b)(a-b)} = \dfrac{a^2+b^2}{2a}$ (5)

k) $\dfrac{x-\frac{1}{x}}{x-\frac{x}{x+\frac{1}{x}}} = \dfrac{x^4-1}{x^2\left(x^2-x+1\right)}$ (4)

l) $$\frac{a}{a+\frac{a}{1-\frac{a}{a-x}}} = \frac{a}{a+\frac{a}{\frac{a-x-a}{a-x}}} = \frac{a}{a-\frac{a(a-x)}{x}} = \frac{a}{\frac{ax-a^2+ax}{x}} = \frac{ax}{2ax-a^2} = \frac{x}{2x-a} \qquad (4)$$

m) $$\frac{1}{k+\frac{1}{k+\frac{1}{k+\frac{1}{k}}}} = \frac{1}{k+\frac{1}{k+\frac{k}{k^2+1}}} = \frac{1}{k+\frac{k^2+1}{k^3+2k}} = \frac{k^3+2k}{k^4+3k^2+1} \qquad (4)$$