

1.6. Prüfungsaufgaben zur Potenzrechnung

Aufgabe 1: Potenzen mit natürlichen Exponenten

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an. (1. P - 3. P = Potenzregeln, 1. bF - 3. bF = binomische Formeln, D = Definition, K = Kürzen)

a) $2 \cdot 2^x$	d) $10^s : 5^s$	g) $\left(\frac{4}{3}\right)^{12} \cdot \left(\frac{3}{2}\right)^{12}$	j) $(a^2 \cdot b^3)^n$	n) $\left(\frac{5a^m b^n}{10p^7 q^3}\right)^{10}$	q) $\frac{a^5 \cdot x^3 - a^7 \cdot x^5}{a^6 \cdot x^4 + a^5 \cdot x^3}$
b) $3^x \cdot 6^x$	e) $2,4^x : (-0,8)^x$	h) $(x^2)^3$	k) $(x^n \cdot y^m)^{n+m}$	o) $\frac{(x^2 - 4)^3}{(x + 2)^3}$	r) $\frac{(a^3 + ab^2)^2}{a^6 - a^2 b^4}$
c) $\left(\frac{2}{3}\right)^n \cdot 6^n$	f) $4,5^k : 3^k$	i) $(5a^2 b^7)^4$	l) $(a^{k-1})^{k+1}$	p) $\frac{(x^2 - 1)^5}{(x - 1)^5}$	s) $\frac{(a - b)^{2n} (a^2 + ab)^n}{(a^2 - b^2)^n a^n}$
			m) $8^{n+1} - 3 \cdot 2^{3n+1}$		t) $\frac{(x^2 - y^2)^n x^n}{(x - y)^{2n} (x^2 + xy)^n}$

Lösungen

$$a) 2 \cdot 2^x \stackrel{D}{=} 2^1 \cdot 2^x \stackrel{1.P}{=} 2^{1+x} \quad (1)$$

$$b) 3^x \cdot 6^x \stackrel{2.P}{=} (3 \cdot 6)^x = 18^x \quad (1)$$

$$c) \left(\frac{2}{3}\right)^n \cdot 6^n \stackrel{2.P}{=} \left(\frac{2}{3} \cdot 6\right)^n = 4^n \quad (1)$$

$$d) 10^s : 5^s \stackrel{2.P}{=} (10 : 5)^s = 2^s \quad (1)$$

$$e) 2,4^x : (-0,8)^x \stackrel{2.P}{=} [2,4 : (-0,8)]^x = [-3]^x \quad (1)$$

$$f) 4,5^k : 3^k \stackrel{2.P}{=} (4,5 : 3)^k = 1,5^k \quad (1)$$

$$g) \left(\frac{4}{3}\right)^{12} \cdot \left(\frac{3}{2}\right)^{12} \stackrel{2.P}{=} 2^{12} \quad (1)$$

$$h) (x^2)^3 \stackrel{3.P}{=} x^{2 \cdot 3} = x^6 \quad (1)$$

$$i) (5a^2 b^7)^4 \stackrel{3.P}{=} 5^4 \cdot a^{2 \cdot 4} \cdot b^{7 \cdot 4} = 625 a^8 b^{28} \quad (1)$$

$$j) (a^2 \cdot b^3)^n \stackrel{3.P}{=} a^{2n} \cdot b^{3n} \quad (1)$$

$$k) (x^n \cdot y^m)^{n+m} \stackrel{3.P}{=} x^{n^2+nm} \cdot y^{mn+m^2} \quad (1)$$

$$l) (a^{k-1})^{k+1} \stackrel{3.P}{=} a^{k^2-1} \quad (1)$$

$$m) 8^{n+1} - 3 \cdot 2^{3n+1} \stackrel{3.P}{=} (2^3)^{n+1} - 3 \cdot 2^{3n+1} = 2^{3n+3} - 3 \cdot 2^{3n+1} = 2^{3n+1} \cdot (2^2 - 3) = 2^{3n+1} \cdot (-1) = -2^{3n+1} \quad (2)$$

$$n) \left(\frac{5a^m b^n}{10p^7 q^3}\right)^{10} \stackrel{3.P}{=} \frac{a^{10m} b^{10n}}{1024p^{70} q^{30}} \quad (1)$$

$$o) \frac{(x^2 - 4)^3}{(x + 2)^3} \stackrel{3.P}{=} \left(\frac{x^2 - 4}{x + 2}\right)^3 \stackrel{3.bF}{=} \left(\frac{(x+2)(x-2)}{x+2}\right)^3 = (x-2)^3 \quad (1)$$

$$p) \frac{(x^2 - 1)^5}{(x - 1)^5} \stackrel{3.P}{=} \left(\frac{x^2 - 1}{x - 1}\right)^5 \stackrel{3.bF}{=} \left(\frac{(x+1)(x-1)}{x-1}\right)^5 = (x+1)^5 \quad (1)$$

$$q) \frac{a^5 \cdot x^3 - a^7 \cdot x^5}{a^6 \cdot x^4 + a^5 \cdot x^3} = \frac{a^5 x^3 (1 - a^2 x^2)}{a^5 x^3 (ax + 1)} \stackrel{3.bF}{=} \frac{(1 - ax)(1 + ax)}{1 + ax} = 1 - ax \quad (3)$$

$$r) \frac{(a^3 + ab^2)^2}{a^6 - a^2 b^4} \stackrel{3.bF}{=} \frac{(a^3 + ab^2)^2}{(a^3 - ab^2)(a^3 + ab^2)} = \frac{a^3 + ab^2}{a^3 - ab^2} \stackrel{T}{=} \frac{a(a^2 + b^2)}{a(a^2 - b^2)} = \frac{a^2 + b^2}{a^2 - b^2} \quad (3)$$

$$s) \frac{(a - b)^{2n} (a^2 + ab)^n}{(a^2 - b^2)^n a^n} \stackrel{3.P, 3.bF}{=} \left(\frac{(a - b)^2 a(a + b)}{(a - b)(a + b)a}\right)^n = (a - b)^n \quad (3)$$

$$t) \frac{(x^2 - y^2)^n x^n}{(x - y)^{2n} (x^2 + xy)^n} \stackrel{3.P}{=} \left(\frac{(x - y)(x + y)x}{(x - y)^2 x(x + y)}\right)^n = (x - y)^{-n} \quad (3)$$

Aufgabe 2: Potenzen mit ganzzahligen Exponenten

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrundeliegende Potenzregel an. (1. P - 3. P = Potenzgesetze, 1. - 3. bF = Binomische Formeln, D = Definition)

a) 2^{-3}	f) $\left(\frac{x}{y}\right)^{-n}$	k) $a^2 : a^6$	p) $\frac{6ax^{-2} - 4x^2}{2x^{-3}}$	u) $(a^4b^{-2} - a^{-2}b^4) : (a^2b^{-1} - a^{-1}b^2)$
b) 5^{-2}	g) $\left(\frac{8}{125}\right)^{-1} : \left(\frac{5}{2}\right)^3$	l) $3 : 3^x$	q) $\frac{6xc^{-2} - 9cy}{3c^{-5}}$	v) $(x^{-4}y^2 - x^2y^{-4}) : (x^{-2}y - xy^{-2})$
c) $\left(\frac{1}{2}\right)^{-3}$	h) $\left(\frac{7}{3^2}\right)^{-2} \cdot \left(\frac{81}{49}\right)^{-2}$	m) $r^{2a} : r^{a-1}$	r) $\frac{(2xy^2)^2}{(4x^2y^2)^3} : \frac{2y^{-3}}{3x^4}$	w) $\frac{(a^2)^3 \cdot (b^3)^{-2} \cdot (ab)^4}{(ab^2)^{-3} \cdot (a^3)^4 \cdot b^5}$
d) $\left(\frac{m}{n}\right)^{-k}$	i) $\left(\frac{p}{q}\right)^{-z} : \left(\frac{p}{2q}\right)^{-z}$	n) $x^{1-y} : x^{y-1}$	s) $\frac{(2x^3y^{-2})^4}{(4y)^5} : \frac{(x^4y^{-3})^3}{4y^5}$	x) $(u^{-3} + v^{-3}) \cdot (u^{-3} - v^{-3})$
e) $\left(\frac{2}{3}\right)^{-2}$	j) $x^4 \cdot x^{-2}$	o) $\frac{x^5 \cdot y^{-5} \cdot z^8}{y^{-3} \cdot z^2 \cdot x^9}$	t) $\frac{a^6}{b^{m+3}} : \frac{a^8}{b^{m+4}}$	y) $\left(\frac{8c^{-5}}{9a^{-3}b^9}\right)^{-3} \cdot \left(\frac{3a^{-3}c^2}{4b^{-5}}\right)^{-5}$
				z) $\left(1 + \frac{2}{p}\right)^2 \cdot \left(\frac{1}{p} - \left(\frac{p}{2} - 1\right)^{-1}\right)^{-2}$

Lösungen

$$a) 2^{-3} \stackrel{D}{=} \frac{1}{2^3} = \frac{1}{8} \quad (1)$$

$$b) 5^{-2} \stackrel{D}{=} \frac{1}{5^2} = \frac{1}{25} \quad (1)$$

$$c) \left(\frac{1}{2}\right)^{-3} \stackrel{D}{=} 2^3 = 8 \quad (1)$$

$$d) \left(\frac{m}{n}\right)^{-k} \stackrel{D}{=} \left(\frac{n}{m}\right)^k \quad (1)$$

$$e) \left(\frac{2}{3}\right)^{-2} \stackrel{D}{=} \left(\frac{3}{2}\right)^2 = \frac{9}{4} \quad (1)$$

$$f) \left(\frac{x}{y}\right)^{-n} \stackrel{D}{=} \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n} \quad (1)$$

$$g) \left(\frac{8}{125}\right)^{-1} : \left(\frac{5}{2}\right)^3 \stackrel{D}{=} \frac{125}{8} : \frac{125}{8} = 1 \quad (1)$$

$$h) \left(\frac{7}{3^2}\right)^{-2} \cdot \left(\frac{81}{49}\right)^{-2} \stackrel{2.P}{=} \left(\frac{7 \cdot 81}{9 \cdot 49}\right)^{-2} = \left(\frac{9}{7}\right)^{-2} \stackrel{D}{=} \frac{49}{81} \quad (1)$$

$$i) \left(\frac{p}{q}\right)^{-z} : \left(\frac{p}{2q}\right)^{-z} \stackrel{D}{=} \left(\frac{q}{p}\right)^z : \left(\frac{p}{2q}\right)^z \stackrel{2.P}{=} \left(\frac{q \cdot p}{p \cdot 2q}\right)^z = \left(\frac{1}{2}\right)^z \stackrel{D}{=} 2^{-z} \quad (1)$$

$$j) x^4 \cdot x^{-2} \stackrel{1.P}{=} x^{4-2} = x^2 \quad (1)$$

$$k) a^2 : a^6 \stackrel{D, 1.P}{=} a^{2-6} = a^{-4} \quad (1)$$

$$l) 3 : 3^x \stackrel{D, 1.P}{=} 3^{1-x} \quad (1)$$

$$m) r^{2a} : r^{a-1} \stackrel{D, 1.P}{=} r^{2a-(a-1)} = r^{1+a} \quad (1)$$

$$n) x^{1-y} : x^{y-1} \stackrel{D, 1.P}{=} x^{1-y-(y-1)} = x^{2-2y} \quad (1)$$

$$o) \frac{x^5 \cdot y^{-5} \cdot z^8}{y^{-3} \cdot z^2 \cdot x^9} \stackrel{D, 1.P}{=} x^{5-9} \cdot y^{-5-(-3)} \cdot z^{8-2} = x^{-4} \cdot y^{-2} \cdot z^6 \quad (1)$$

$$p) \frac{6ax^{-2} - 4x^2}{2x^{-3}} = \frac{6ax^{-2}}{2x^{-3}} - \frac{4x^2}{2x^{-3}} \stackrel{D, 1.P}{=} 3ax - 2x^5 \quad (2)$$

$$q) \frac{6xc^{-2} - 9cy}{3c^{-5}} = \frac{6xc^{-2}}{3c^{-5}} - \frac{9cy}{3c^{-5}} \stackrel{D, 1.P}{=} 2xc^3 - 3cy^6 \quad (2)$$

$$r) \frac{(2xy^2)^2}{(4x^2y^2)^3} : \frac{2y^{-3}}{3x^4} \stackrel{3.P}{=} \frac{2^2 x^2 y^4}{4^3 x^6 y^6} \cdot \frac{3x^4}{2y^{-3}} \stackrel{1.P}{=} \frac{4 \cdot 3 \cdot x^{2+4-6} \cdot y^{4-6-(-3)}}{4^3 \cdot 2} = \frac{3}{32} y \quad (3)$$

$$s) \frac{(2x^3y^{-2})^4}{(4y)^5} : \frac{(x^4y^{-3})^3}{4y^5} \stackrel{3.P}{=} \frac{2^4 x^{12} y^{-8}}{4^5 y^5} \cdot \frac{4y^5}{x^{-12} y^{-9}} \stackrel{1.P}{=} \frac{2^4 \cdot 4 \cdot x^{12-12} y^{-8+5-(-9)-5}}{4^5} = \frac{1}{16} y \quad (3)$$

$$t) \frac{a^6}{b^{m+3}} : \frac{a^8}{b^{m+4}} \stackrel{D, 1.P}{=} a^6 b^{m+3} \cdot a^{-8} b^{-8-(m+4)} \stackrel{1.P}{=} a^{6-8} b^{m+3+m+4} = a^{-2} b^{2m+7} \quad (2)$$

$$u) (a^4 b^{-2} - a^{-2} b^4) : (a^2 b^{-1} - a^{-1} b^2) = \frac{\frac{a^4}{b^2} - \frac{b^4}{a^2}}{\frac{a^2}{b} - \frac{b^2}{a}} \stackrel{3.bF}{=} \frac{\left(\frac{a^2}{b} - \frac{b^2}{a}\right) \cdot \left(\frac{a^2}{b} + \frac{b^2}{a}\right)}{\frac{a^2}{b} - \frac{b^2}{a}} = \frac{a^2}{b} + \frac{b^2}{a} \quad (3)$$

$$v) (x^{-4} y^2 - x^2 y^{-4}) : (x^{-2} y - x y^{-2}) = \frac{\frac{y^2}{x^4} - \frac{x^2}{y^4}}{\frac{y^2}{x} - \frac{x}{y^2}} \stackrel{3.bF}{=} \frac{\left(\frac{y^2}{x} - \frac{x}{y^2}\right) \cdot \left(\frac{y^2}{x} + \frac{x}{y^2}\right)}{\frac{y^2}{x} - \frac{x}{y^2}} = \frac{y^2}{x} + \frac{x}{y^2} \quad (3)$$

$$w) \frac{(a^2)^3 \cdot (b^3)^{-2} \cdot (ab)^4}{(ab^2)^{-3} \cdot (a^3)^4 \cdot b^5} \stackrel{2.+3.P}{=} \frac{a^6 \cdot b^{-6} \cdot a^4 b^4}{a^{-3} b^{-6} \cdot a^{12} \cdot b^5} \stackrel{1.P}{=} a^{6-4-(-3)-12} \cdot b^{-6+4-(-6)-5} = \frac{a}{b} \quad (3)$$

$$x) (u^{-3} + v^{-3}) \cdot (u^{-3} - v^{-3}) \stackrel{3.bF}{=} (u^{-3})^2 - (v^{-3})^2 \stackrel{3.P}{=} u^{-6} - v^{-6} \quad (2)$$

$$y) \left(\frac{8c^{-5}}{9a^{-3}b^9}\right)^{-3} \cdot \left(\frac{3a^{-3}c^2}{4b^{-5}}\right)^{-5} \stackrel{3.P}{=} \frac{8^{-3} c^{15}}{9^{-3} a^9 b^{-27}} \cdot \frac{3^{-5} a^{15} c^{-10}}{4^{-5} b^{25}} \stackrel{1.P}{=} \frac{2^{3 \cdot (-3)} \cdot 3^{-5} \cdot c^{15-10} \cdot a^{15-9}}{3^{2 \cdot (-3)} \cdot 2^{2 \cdot (-5)} \cdot b^{-27+25}}$$

$$\stackrel{1.P}{=} \frac{2^{-9+10} \cdot 3^{-5+6} \cdot c^5 \cdot a^6}{b^{-2}} \stackrel{1.P}{=} 6 \cdot a^6 b^2 c^5 \quad (2)$$

$$z) \left(1 + \frac{2}{p}\right)^2 \cdot \left(\frac{1}{p} - \left(\frac{p}{2} - 1\right)^{-1}\right)^{-2} = \left(\frac{p+2}{p}\right)^2 \cdot \left(\frac{1}{p} - \left(\frac{p-2}{2}\right)^{-1}\right)^{-2} \quad (1)$$

$$\stackrel{3.P}{=} \left(\frac{p+2}{p}\right)^2 \cdot \left(\frac{1}{p} - \frac{2}{p-2}\right)^{-2} \quad (1)$$

$$= \left(\frac{p+2}{p}\right)^2 \cdot \left(\frac{p-2-2p}{p \cdot (p-2)}\right)^{-2} \quad (1)$$

$$= \left(\frac{p+2}{p}\right)^2 \cdot \left(\frac{p(p-2)}{-p-2}\right)^2 \quad (1)$$

$$= \left(\frac{p+2}{p} \cdot \frac{p(p-2)}{(-1)(p+2)}\right)^2 = \left(\frac{p-2}{-1}\right)^2 = (p-2)^2 \quad (1)$$

Aufgabe 3: n-te Wurzel

Gib das Ergebnis an und begründe:

a) $\sqrt[4]{256}$ b) $\sqrt[5]{243}$ c) $\left(\sqrt[3]{6}\right)^6$ d) $\left(\sqrt[4]{3}\right)^8$

Lösungen:

$$\sqrt[4]{256} = 4, \text{ weil } 4^4 = 256 \quad (2)$$

$$\sqrt[5]{243} = 3, \text{ weil } 3^5 = 243 \quad (2)$$

$$\left(\sqrt[3]{6}\right)^6 \stackrel{3.P}{=} \left(\left(\sqrt[3]{6}\right)^3\right)^2 = 6^2 = 36 \quad (3)$$

$$\left(\sqrt[4]{3}\right)^8 \stackrel{3.P}{=} \left(\left(\sqrt[4]{3}\right)^4\right)^2 = 3^2 = 9 \quad (3)$$

Aufgabe 4: Potenzen mit rationalen Exponenten: nur 3. Potenzregel/Definition

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an.

(1. - 3. P = Potenzgesetze, 1. - 3. bF = Binomische Formeln, D = Definition)

a) $(35^0 \cdot 8^{0.5} \cdot 4^1)^2$ f) $25^{\frac{3}{2}}$ k) $\sqrt[5]{2}^{-10}$ p) $\sqrt[3]{x^2}^6$ u) $\left(a^{-\frac{3}{2}}\right)^{-\frac{4}{3}}$

$$\begin{array}{llllll}
\text{b) } (6^{-4} \cdot 3^6)^{0,5} & \text{g) } \left(\frac{1}{\sqrt[3]{5}}\right)^2 & \text{l) } \sqrt[4]{x}^{-2} & \text{q) } \sqrt[5]{y^3}^{10} & \text{v) } \left(x^{-\frac{5}{4}} \cdot y^{-\frac{5}{8}}\right)^{\frac{4}{5}} \\
\text{c) } 81^{\frac{1}{4}} & \text{h) } \left(\frac{27}{64}\right)^{\frac{2}{3}} & \text{m) } \sqrt[3]{x}^6 & \text{r) } \sqrt[3]{s^4}^{3n} & \text{w) } \sqrt{y \cdot \sqrt{y} \cdot \sqrt{y}} \\
\text{d) } 125^{\frac{1}{3}} & \text{i) } \left(\frac{27}{8}\right)^{-\frac{2}{3}} & \text{n) } \sqrt[2]{y^4} & \text{s) } \sqrt[6]{\sqrt{x}^3} & \text{x) } \frac{\sqrt[5]{f}}{\sqrt[5]{f} \cdot \sqrt{f^3}} \\
\text{e) } 8^{\frac{2}{3}} & \text{j) } \sqrt[3]{5}^6 & \text{o) } \sqrt[4]{x} & \text{t) } \left(\sqrt{\sqrt{b^3}}\right)^{4n} & &
\end{array}$$

Lösungen

$$\text{a) } (35^0 \cdot 8^{0,5} \cdot 4^1)^2 \stackrel{\text{D, 2.P}}{=} 35^0 \cdot 8^1 \cdot 4^2 \stackrel{\text{D}}{=} 2 \cdot 4^3 = 128 \quad (1)$$

$$\text{b) } (6^{-4} \cdot 3^6)^{0,5} \stackrel{\text{D, 2.P}}{=} 6^{-2} \cdot 3^3 \stackrel{\text{D}}{=} \frac{27}{36} = \frac{3}{4} \quad (1)$$

$$\text{c) } 81^{\frac{1}{4}} \stackrel{\text{D}}{=} \sqrt[4]{81} = 3 \quad (1)$$

$$\text{d) } 125^{\frac{1}{3}} \stackrel{\text{D}}{=} \sqrt[3]{125} = 5 \quad (1)$$

$$\text{e) } 8^{\frac{2}{3}} \stackrel{\text{3.P}}{=} \left(8^{\frac{1}{3}}\right)^2 \stackrel{\text{D}}{=} 2^2 = 4 \quad (1)$$

$$\text{f) } 25^{\frac{3}{2}} \stackrel{\text{3.P}}{=} \left(25^{\frac{1}{2}}\right)^3 \stackrel{\text{D}}{=} 5^3 = 125 \quad (1)$$

$$\text{g) } \left(\frac{1}{\sqrt[3]{5}}\right)^2 \stackrel{\text{D}}{=} \left(5^{-\frac{1}{3}}\right)^2 \stackrel{\text{3.P}}{=} 5^{-\frac{1}{3} \cdot 2} \stackrel{\text{3.P}}{=} 5^{-\frac{2}{3}} \stackrel{\text{D}}{=} 5^2 \cdot \frac{1}{5} = 25 \cdot \frac{1}{5} = \frac{1}{\sqrt[3]{25}} \quad (1)$$

$$\text{h) } \left(\frac{27}{64}\right)^{\frac{2}{3}} \stackrel{\text{3.P}}{=} \left(\left(\frac{27}{64}\right)^{\frac{1}{3}}\right)^2 \stackrel{\text{D}}{=} \left(\frac{3}{4}\right)^2 = \frac{9}{16} \quad (1)$$

$$\text{i) } \left(\frac{27}{8}\right)^{-\frac{2}{3}} \stackrel{\text{D}}{=} \left(\frac{8}{27}\right)^{\frac{2}{3}} \stackrel{\text{3.P}}{=} \left(\left(\frac{8}{27}\right)^{\frac{1}{3}}\right)^2 \stackrel{\text{D}}{=} \left(\frac{2}{3}\right)^2 = \frac{4}{9} \quad (1)$$

$$\text{j) } \sqrt[3]{5}^6 \stackrel{\text{D, 3.P}}{=} 5^{\frac{6}{3}} = 25 \quad (1)$$

$$\text{k) } \sqrt[5]{2}^{-10} \stackrel{\text{D, 3.P}}{=} 2^{-\frac{10}{5}} \stackrel{\text{D}}{=} \frac{1}{2} \quad (1)$$

$$\text{l) } \sqrt[4]{x}^{-2} \stackrel{\text{D, 3.P}}{=} x^{-\frac{2}{4}} \stackrel{\text{D}}{=} \frac{1}{\sqrt{x}} \quad (1)$$

$$\text{m) } \sqrt[3]{x}^6 \stackrel{\text{D}}{=} \left(x^{\frac{1}{3}}\right)^6 \stackrel{\text{3.P}}{=} x^{\frac{1}{3} \cdot 6} = x^2 \quad (1)$$

$$\text{n) } \sqrt[2]{y^4} \stackrel{\text{D}}{=} y^{4 \cdot \frac{1}{2}} \stackrel{\text{3.P}}{=} y^2 = y^2 \quad (1)$$

$$\text{o) } \sqrt[4]{x} \stackrel{\text{D}}{=} \left(x^{\frac{1}{4}}\right)^{\frac{1}{2}} \stackrel{\text{3.P}}{=} x^{\frac{1}{4} \cdot \frac{1}{2}} \stackrel{\text{D}}{=} \sqrt[8]{x} \quad (1)$$

$$\text{p) } \sqrt[3]{x^2}^6 \stackrel{\text{D, 3.P}}{=} x^{2 \cdot \frac{1}{3} \cdot 6} = x^4 \quad (1)$$

$$\text{q) } \sqrt[5]{y^3}^{10} \stackrel{\text{D, 3.P}}{=} y^{3 \cdot \frac{1}{5} \cdot 10} = y^6 \quad (1)$$

$$\text{r) } \sqrt[3]{s^4}^{3n} \stackrel{\text{D, 3.P}}{=} s^{4 \cdot \frac{1}{3} \cdot 3n} = s^{4n} \quad (1)$$

$$\text{s) } \sqrt[6]{\sqrt{x}^3} \stackrel{\text{D, 3.P}}{=} x^{\frac{1}{2} \cdot \frac{1}{6} \cdot 3} = x^{\frac{1}{4}} \quad (2)$$

$$t) \left(\sqrt{\sqrt{b^3}} \right)^{4n} \stackrel{D, 3.P}{=} b^{3 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 4n} = b^{3n} \quad (2)$$

$$u) \left(a^{\frac{3}{2}} \right)^{-\frac{4}{3}} \stackrel{3.P}{=} a^{\left(\frac{3}{2} \right) \left(-\frac{4}{3} \right)} = a^2 \quad (1)$$

$$v) \left(x^{\frac{5}{4}} \cdot y^{\frac{5}{8}} \right)^{-\frac{4}{5}} \stackrel{3.P}{=} x^{\left(\frac{5}{4} \right) \left(-\frac{4}{5} \right)} \cdot y^{\left(\frac{5}{8} \right) \left(-\frac{4}{5} \right)} = xy^{0,5} \quad (1)$$

$$w) \sqrt{y \cdot \sqrt{y \cdot \sqrt{y}}} \stackrel{D}{=} \left(y \cdot (y \cdot y^{0,5})^{0,5} \right)^{0,5} \stackrel{1.P}{=} \left(y \cdot (y^{1,5})^{0,5} \right)^{0,5} \stackrel{3.P}{=} \left(y \cdot y^{0,75} \right)^{0,5} \stackrel{1.P}{=} \left(y^{1,75} \right)^{0,5} \stackrel{3.P}{=} y^{0,875} \quad (4)$$

$$x) \frac{\sqrt[5]{f} \cdot \sqrt{f}}{\sqrt[3]{f} \cdot \sqrt{f^3}} \stackrel{D}{=} \frac{\left(f^{\frac{1}{5}} \right)^{\frac{1}{2}}}{f^{\frac{1}{3}} \cdot f^{\frac{3}{2}}} \stackrel{3.P}{=} \frac{f^{\frac{1}{10}}}{f^{\frac{1}{3} + \frac{3}{2}}} \stackrel{1.P}{=} f^{\frac{1}{10} - \frac{17}{10}} = f^{-\frac{16}{10}} = f^{-\frac{8}{5}} \quad (4)$$

Aufgabe 5: Potenzen mit rationalen Exponenten: nur 1. Potenzregel

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an. (1. - 3. P = Potenzgesetze, 1. - 3. bF = binomische Formeln, D = Definition)

$$a) 3^{\frac{1}{4}} \cdot 9^{\frac{1}{4}} \cdot 3^{\frac{1}{4}}$$

$$f) (3x^2 + 2x^{\frac{2}{3}}) \cdot 4x^{\frac{3}{2}}$$

$$k) \frac{\sqrt[3]{2y} : \sqrt[3]{y}}{\sqrt{y} : \sqrt[4]{y}}$$

$$b) 2^{\frac{1}{3}} \cdot 4^{\frac{2}{3}}$$

$$g) 3a^4 b^{\frac{2}{3}} \cdot (a - 3b^2)$$

$$l) \frac{\sqrt{t} : \sqrt[3]{t}}{t}$$

$$c) 5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}}$$

$$h) (6k^4 m^5 n^3 + 12k^2 m^2 n^{\frac{1}{4}}) : (3k^{-2} m^2 n^{\frac{1}{2}})$$

$$m) a\sqrt[3]{b} + b\sqrt[3]{a} : \sqrt[3]{ab}$$

$$d) a^{\frac{3}{2}} \cdot a^{\frac{5}{2}}$$

$$i) \sqrt[3]{2} \cdot \sqrt[3]{4}$$

$$e) \left(a^{\frac{m}{3}} \cdot a^{\frac{m}{6}} \right) : a^{\frac{m}{4}}$$

$$j) \sqrt[3]{x} \cdot \sqrt{x}$$

Lösungen

$$a) 3^{\frac{1}{4}} \cdot 9^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} = 3^{\frac{1}{4}} \cdot 3^{\frac{2}{4}} \cdot 3^{\frac{1}{4}} \stackrel{1.P}{=} 3^{\frac{1+2+1}{4}} = 3 \quad (2)$$

$$b) 2^{\frac{1}{3}} \cdot 4^{\frac{2}{3}} = 2^{\frac{1}{3}} \cdot 2^{\frac{2 \cdot 2}{3}} \stackrel{1.P}{=} 2^{\frac{1+2 \cdot 2}{3}} = 2^{\frac{5}{3}} \quad (2)$$

$$c) 5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}} = 5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}} \stackrel{1.P}{=} 5^{\frac{1+2}{3}} = 5 \quad (2)$$

$$d) a^{\frac{3}{2}} \cdot a^{\frac{5}{2}} \stackrel{1.P}{=} a^{\frac{3+5}{2}} = a^4 \quad (2)$$

$$e) \left(a^{\frac{m}{3}} \cdot a^{\frac{m}{6}} \right) : a^{\frac{m}{4}} \stackrel{1.P}{=} a^{\frac{m}{3} + \frac{m}{6} - \frac{m}{4}} = a^{\frac{m}{4}} \quad (2)$$

$$f) (3x^2 + 2x^{\frac{2}{3}}) \cdot 4x^{\frac{3}{2}} \stackrel{1.P}{=} 3 \cdot 4 \cdot x^{\frac{1}{2} + \frac{3}{2}} + 2 \cdot 4 \cdot x^{\frac{2}{3} + \frac{3}{2}} = 12x^2 + 8x^{\frac{13}{6}} \quad (2)$$

$$g) 3a^4 b^{\frac{2}{3}} \cdot (a - 3b^2) \stackrel{1.P}{=} 3 \cdot a^{\frac{1}{4} + 1} b^{\frac{2}{3}} - 3 \cdot 3 \cdot a^4 b^{\frac{2}{3} + 2} = 3a^4 b^{\frac{2}{3}} - 9a^4 b^{\frac{8}{3}} \quad (2)$$

$$h) (6k^4 m^5 n^3 + 12k^2 m^2 n^{\frac{1}{4}}) : (3k^{-2} m^2 n^{\frac{1}{2}}) \stackrel{1.P}{=} 2k^{\frac{3}{4} + 2} m^{5-2} n^{3 - \frac{1}{2}} + 4k^{2+2} m^{\frac{1}{2} - 2} n^{\frac{1}{4} - \frac{1}{2}} = 2k^{\frac{11}{4}} m^3 n^{\frac{5}{2}} + 4k^4 m^{-\frac{3}{2}} n^{-\frac{1}{4}} \quad (2)$$

$$i) \sqrt[3]{2} \cdot \sqrt[3]{4} \stackrel{D}{=} 2^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} \stackrel{1.P}{=} 2^{\frac{1+2}{3}} = 2 \quad (2)$$

$$j) \sqrt[3]{x} \cdot \sqrt{x} \stackrel{1.P}{=} x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} \stackrel{1.P}{=} x^{\frac{1}{3} + \frac{1}{2}} = x^{\frac{5}{6}} \quad (1)$$

$$k) \frac{\sqrt[3]{2y} : \sqrt[3]{y}}{\sqrt{y} : \sqrt[4]{y}} \stackrel{D, 1.P}{=} \frac{2^{\frac{1}{3}} \cdot y^{\frac{1}{3} - \frac{1}{3}}}{y^{\frac{1}{2}} : y^{-\frac{1}{4}}} = 2^{\frac{1}{3}} \cdot y^{-\frac{1}{4}} \quad (2)$$

$$l) \frac{\sqrt{t} : \sqrt[3]{t}}{t} \stackrel{D, 1.P}{=} \frac{t^{\frac{1}{2} - \frac{1}{3}}}{t^1} = t^{-\frac{5}{6}} \quad (2)$$

$$m) a\sqrt[3]{b} + b\sqrt[3]{a} : \sqrt[3]{ab} \stackrel{D}{=} \left(ab^{\frac{1}{3}} + a^{\frac{1}{3}} b \right) : \left(ab^{\frac{1}{3}} \right) \stackrel{1.P}{=} a^{1 - \frac{1}{3}} b^{\frac{1}{3} - \frac{1}{3}} + a^{\frac{1}{3} - \frac{1}{3}} b^{1 - \frac{1}{3}} = a^{\frac{2}{3}} + b^{\frac{2}{3}} \quad (2)$$

Aufgabe 6: Potenzen mit rationalen Exponenten: 1. und 3. Potenzregel

Schreibe als Potenzen mit gleichen Basen und vereinfache:

a) $2 \cdot \left(\frac{1}{8}\right)^{\frac{1}{5}}$	e) $x \cdot \left(\frac{x}{y}\right)^{\frac{1}{3}}$	i) $\sqrt[3]{x^2} \cdot \sqrt[3]{x^4}$	m) $\sqrt[3]{x^4} : \sqrt[4]{x^5}$
b) $ab^2 \cdot \left(\frac{4}{a^2b^3}\right)^{\frac{1}{3}}$	f) $(x-y) \cdot x^2 - y^2^{-\frac{1}{2}}$	j) $\sqrt[5]{a} \cdot \sqrt[5]{a^4}$	n) $\frac{\sqrt{b} \cdot \sqrt[3]{b}}{\sqrt[4]{b^3}}$
c) $6 \cdot \left(\frac{7}{36}\right)^{\frac{1}{3}}$	g) $\sqrt[2]{a^3} \cdot \sqrt[2]{a^3}$	k) $\sqrt{\sqrt[3]{16} \cdot \sqrt[9]{64}}$	o) $\frac{x}{\sqrt[3]{x^2} \cdot \sqrt[4]{x}}$
d) $(a+b) \cdot \left(\frac{5}{a+b}\right)^{\frac{1}{2}}$	h) $\sqrt[3]{a^2} \cdot \sqrt[3]{a^2}$	l) $\sqrt[7]{5^5} : \sqrt[3]{5^2}$	p) $\frac{\sqrt[6]{a^5}}{\sqrt{a} : \sqrt[3]{a}}$
			q) $\sqrt[3]{a^5b} \cdot \sqrt[3]{ab^2}$

Lösungen

a) $2 \cdot \left(\frac{1}{8}\right)^{\frac{1}{5}} = 2 \cdot \frac{1}{2} = 1$ (2)

b) $ab^2 \cdot \left(\frac{4}{a^2b^3}\right)^{\frac{1}{3}} = a^{\frac{1}{3}} b^{\frac{2}{3}} \cdot \frac{4^{\frac{1}{3}}}{a^{\frac{2}{3}} b^{\frac{1}{3}}} = \frac{4^{\frac{1}{3}} a^{\frac{1}{3}-\frac{2}{3}} b^{\frac{2}{3}-\frac{1}{3}}}{1} = \frac{4^{\frac{1}{3}}}{a^{\frac{1}{3}}} b^{\frac{1}{3}}$ (2)

c) $6 \cdot \left(\frac{7}{36}\right)^{\frac{1}{3}} = 6 \cdot \frac{7^{\frac{1}{3}}}{36^{\frac{1}{3}}} = \frac{6 \cdot 7^{\frac{1}{3}}}{6 \cdot 6^{\frac{1}{3}}} = \frac{7^{\frac{1}{3}}}{6^{\frac{1}{3}-1}} = \frac{7^{\frac{1}{3}}}{6^{\frac{2}{3}}}$ (2)

d) $(a+b) \cdot \left(\frac{5}{a+b}\right)^{\frac{1}{2}} = (a+b)^{1-\frac{1}{2}} \cdot 5^{\frac{1}{2}} = (a+b)^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5(a+b)^{\frac{1}{2}}$ (2)

e) $x \cdot \left(\frac{x}{y}\right)^{\frac{1}{3}} = x^{\frac{1}{3}} \cdot \frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}} = \frac{x^{\frac{1}{3}+\frac{1}{3}}}{y^{\frac{1}{3}}} = \frac{x^{\frac{2}{3}}}{y^{\frac{1}{3}}}$ (2)

f) $(x-y) \cdot x^2 - y^2^{-\frac{1}{2}} = (x-y) \cdot \frac{1}{x^2 - y^2} = \frac{x-y}{(x-y)(x+y)} = \frac{1}{x+y}$ (2)

g) $\sqrt[2]{a^3} \cdot \sqrt[2]{a^3} = a^{\frac{3}{2}} \cdot a^{\frac{3}{2}} = a^3$ (1)

h) $\sqrt[3]{a^2} \cdot \sqrt[3]{a^2} = a^{\frac{2}{3}} \cdot a^{\frac{2}{3}} = a^{\frac{4}{3}}$ (1)

i) $\sqrt[3]{x^2} \cdot \sqrt[3]{x^4} = x^{\frac{2}{3}} \cdot x^{\frac{4}{3}} = x^{\frac{2}{3}+\frac{4}{3}} = x^2$ (2)

j) $\sqrt[5]{a} \cdot \sqrt[5]{a^4} = \sqrt[5]{a \cdot a^4} = \sqrt[5]{a^5} = a$ (1)

k) $\sqrt{\sqrt[3]{16} \cdot \sqrt[9]{64}} = \left(4^{\frac{2}{3}} \cdot 4^{\frac{1}{9}}\right)^{\frac{1}{2}} \stackrel{(1)}{=} \left(4^{\frac{2}{3}+\frac{1}{9}}\right)^{\frac{1}{2}} = 4^{1 \cdot \frac{1}{2}} = 2$ (1)

l) $\sqrt[7]{5^5} : \sqrt[3]{5^2} = 5^{\frac{5}{7}} \cdot 5^{-\frac{2}{3}} \stackrel{(1)}{=} 5^{\frac{5}{7}-\frac{2}{3}} = 5^{\frac{15-14}{21}} = 5^{\frac{1}{21}}$ (1)

m) $\sqrt[3]{x^4} : \sqrt[4]{x^5} = x^{\frac{4}{3}} \cdot x^{-\frac{5}{4}} \stackrel{(1)}{=} x^{\frac{4}{3}-\frac{5}{4}} = x^{\frac{16-15}{12}} = x^{\frac{1}{12}}$ (1)

n) $\frac{\sqrt{b} \cdot \sqrt[3]{b}}{\sqrt[4]{b^3}} \stackrel{(1)}{=} \frac{b^{\frac{1}{2}} \cdot b^{\frac{1}{3}}}{b^{\frac{3}{4}}} = b^{\frac{1}{2}+\frac{1}{3}-\frac{3}{4}} = b^{\frac{6+4-9}{12}} = b^{\frac{1}{12}}$ (2)

o) $\frac{x}{\sqrt[3]{x^2} \cdot \sqrt[4]{x}} \stackrel{(1)}{=} x^{1-\frac{2}{3}-\frac{1}{4}} = x^{\frac{12-8-3}{12}} = x^{\frac{1}{12}}$ (2)

p) $\frac{\sqrt[6]{a^5}}{\sqrt{a} : \sqrt[3]{a}} \stackrel{(1)}{=} a^{\frac{5}{6}-\left(\frac{1}{2}-\frac{1}{3}\right)} = a^{\frac{5}{6}-\frac{1}{6}} = a^{\frac{4}{6}} = a^{\frac{2}{3}}$ (2)

q) $\sqrt[3]{a^5b} \cdot \sqrt[3]{ab^2} = a^{\frac{5}{3}} b^{\frac{1}{3}} \cdot a^{\frac{1}{3}} b^{\frac{2}{3}} = a^{\frac{5}{3}+\frac{1}{3}} b^{\frac{1}{3}+\frac{2}{3}} = a^2 b$ (2)

Aufgabe 7: Potenzen mit rationalen Exponenten: 1. - 3. Potenzregel

Vereinfache die folgenden Ausdrücke soweit wie möglich. Gib bei jedem Rechenschritt die zugrunde liegende Potenzregel an. (1. - 3-P = Potenzregeln, 1. - 3- bF = binomische Formeln, D = Definition)

a) $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$

h) $\sqrt[4]{\frac{x^8 y}{y^{-17} x^6}}$

o) $\left(x + (x^2 - a^2)^{\frac{1}{2}}\right)^3 \cdot \left(x - (x^2 - a^2)^{\frac{1}{2}}\right)^3$

b) $8^{\frac{1}{2}} : 2^{\frac{1}{2}}$

i) $\sqrt[n]{\frac{x^2 y^{n+1}}{z^{n+1}}} \cdot \sqrt[n]{\frac{z x^{n-2}}{y}}$

p) $\left((x+1) - 2x^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left((x+1) + 2x^{\frac{1}{2}}\right)^{\frac{1}{2}}$

c) $2^{\frac{3}{2}} \cdot 18^{\frac{3}{2}}$

j) $\left(\frac{a^{-\frac{r}{s}} \cdot b^{\frac{r}{s}}}{a \cdot b^{\frac{r}{s}}}\right)^{-\frac{1}{2}}$

q) $a^{-b} \sqrt{x^{a^2-2ab+b^2}} : a^{+b} \sqrt{x^{a^2+2ab+b^2}}$

d) $\sqrt[6]{4a^8 b^4} \cdot \sqrt[6]{16a^{16} b^{14}}$

k) $\frac{\sqrt[3]{x^2}^2 \cdot x^{\frac{1}{6}}}{\sqrt[5]{x^2} \cdot \sqrt[10]{x}}$

r) $(\sqrt{a} + 1)(1 - \sqrt[4]{a})(1 + \sqrt[4]{a}) + a$

e) $\sqrt[7]{(x^4 y)^2 z^5} \cdot \sqrt[7]{x^6 (y^6 z)^2}$

l) $\frac{\sqrt[3]{y} \cdot 2\sqrt[3]{z^3}}{2\sqrt[3]{y^3} \cdot \sqrt[3]{z}}$

e) $(25^{\frac{1}{3}} - 4^{\frac{1}{3}}) \cdot (5^{\frac{1}{3}} + 2^{\frac{1}{3}})$

f) $\sqrt{a^4 a} \cdot \sqrt{a^4 a}$

m) $\left(\frac{2x^2}{3y^{\frac{3}{8}}}\right) : \left(\frac{9x^{-3}}{4y^{\frac{3}{4}}}\right)$

g) $\frac{1}{a \cdot \sqrt[5]{a^2} \cdot \sqrt{a}}$

n) $\frac{\sqrt[3]{a-b^4}}{\sqrt[3]{a^2-b^2}^2}$

Lösungen

a) $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}} \stackrel{2.P}{=} 27^{\frac{1}{3}} = 3$ (1)

b) $8^{\frac{1}{2}} : 2^{\frac{1}{2}} \stackrel{2.P}{=} 4^{\frac{1}{2}} = 2$ (1)

c) $2^{\frac{3}{2}} \cdot 18^{\frac{3}{2}} \stackrel{2.P}{=} 36^{\frac{3}{2}} = 6^3 = 216$ (1)

d) $\sqrt[6]{4a^8 b^4} \cdot \sqrt[6]{16a^{16} b^{14}} \stackrel{1.+2.P}{=} \sqrt[6]{64a^{24} b^{18}} \stackrel{3.P}{=} 2a^4 b^3$ (2)

e) $\sqrt[7]{(x^4 y)^2 z^5} \cdot \sqrt[7]{x^6 (y^6 z)^2} \stackrel{1.+2.P}{=} \sqrt[7]{x^{14} y^{14} z^7} \stackrel{3.P}{=} x^2 y^2 z$ (2)

f) $\sqrt{a^4 a} \cdot \sqrt{a^4 a} \stackrel{2.P}{=} \sqrt{a^4 a \cdot a^4 a} \stackrel{D}{=} \left(a^2 \cdot a^4 \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}} \stackrel{1.+3.P}{=} a^{\frac{11}{8}}$ (3)

g) $\frac{1}{a \cdot \sqrt[5]{a^2} \cdot \sqrt{a}} \stackrel{D}{=} a^{-1} \cdot \left(a^2 \cdot a^{\frac{1}{2}}\right)^{-\frac{1}{5}} \stackrel{1.+3.P}{=} a^{-1} \cdot a^{\frac{5}{2} \cdot \left(-\frac{1}{5}\right)} \stackrel{1.P}{=} a^{-\frac{3}{2}}$ (3)

h) $\sqrt[4]{\frac{x^8 y}{y^{-17} x^6}} \stackrel{1.P}{=} \sqrt[4]{x^2 y^{18}} \stackrel{2.+3.P}{=} x^{\frac{1}{2}} \cdot y^{\frac{9}{2}} \stackrel{2.+3.P}{=} \sqrt{x \cdot y^9}$ (3)

i) $\sqrt[n]{\frac{x^2 y^{n+1}}{z^{n+1}}} \cdot \sqrt[n]{\frac{z x^{n-2}}{y}} \stackrel{1.+2.P}{=} \sqrt[n]{\frac{x^n y^n}{z^n}} \stackrel{2.+3.P}{=} \frac{xy}{z}$ (3)

j) $\left(\frac{a^{-\frac{r}{s}} \cdot b^{\frac{r}{s}}}{a \cdot b^{\frac{r}{s}}}\right)^{-\frac{1}{2}} \stackrel{1.+3.P}{=} \left(a^{-\frac{r}{s} \cdot \frac{r}{s}} \cdot b^{\frac{r}{s} \cdot \frac{r}{s}}\right)^{-\frac{1}{2}} \stackrel{3.P}{=} a^{\left(-\frac{r}{s}\right) \cdot \left(-\frac{1}{2}\right)} = a^{\frac{r}{s}}$ (3)

$$k) \frac{\sqrt[3]{x^2}^2 \cdot x^{\frac{1}{6}}}{\sqrt[5]{x^2} \cdot \sqrt[10]{x}} \stackrel{D}{=} \frac{\left(\frac{2}{x^3}\right)^2 \cdot x^{\frac{1}{6}}}{\frac{2}{x^5} \cdot x^{\frac{1}{10}}} \stackrel{1.+3.P}{=} x^{\frac{4}{3} + \frac{1}{6} - \frac{2}{5} - \frac{1}{10}} = x \quad (3)$$

$$l) \frac{\sqrt[3]{y} \cdot 2\sqrt[3]{z^3}}{2\sqrt[3]{y^3} \cdot \sqrt[3]{z}} \stackrel{D}{=} \frac{\frac{1}{y^{\frac{1}{3}}} \cdot z^{\frac{3}{2x}}}{\frac{3}{y^{2x}} \cdot z^{\frac{1}{x}}} \stackrel{1.P}{=} y^{\frac{1}{3} - \frac{3}{2x}} \cdot z^{\frac{1}{x} - \frac{3}{x}} \stackrel{3.P}{=} 2\sqrt[3]{\frac{z}{y}} \quad (3)$$

$$m) \left(\frac{2x^2}{3y^{\frac{3}{8}}}\right) : \left(\frac{9x^{-3}}{4y^4}\right) = \frac{2x^2 \cdot 4y^4}{3y^{\frac{3}{8}} \cdot 9x^{-3}} \stackrel{1.P}{=} \frac{8}{27} x^{2+3} y^{4 - \frac{3}{8}} = \frac{8}{27} x^5 y^{\frac{3}{8}} \quad (2)$$

$$n) \frac{\sqrt[3]{a-b^4}}{\sqrt[3]{a^2-b^2}} \stackrel{3.P}{=} \frac{\sqrt[3]{a-b^4}}{\sqrt[3]{a^2-b^2}^2} \stackrel{2.P+3.bF}{=} \sqrt[3]{\frac{a-b^4}{a-b^2 \cdot a+b^2}} = \sqrt[3]{\frac{a-b^2}{a+b^2}} \stackrel{2.P}{=} \left(\frac{a-b}{a+b}\right)^{\frac{2}{3}} \quad (3)$$

$$o) \left(x + (x^2 - a^2)^{\frac{1}{2}}\right)^3 \cdot \left(x - (x^2 - a^2)^{\frac{1}{2}}\right)^3 \stackrel{2.P}{=} \left(\left(x + (x^2 - a^2)^{\frac{1}{2}}\right) \cdot \left(x - (x^2 - a^2)^{\frac{1}{2}}\right)\right)^3 \stackrel{2.P}{=} \left(\left(x^2 - (x^2 - a^2)^{\frac{1}{2} \cdot 2}\right)\right)^3$$

$$= x^2 - (x^2 - a^2) \stackrel{3.P}{=} a^6 \quad (4)$$

$$p) \left((x+1) - 2x^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left((x+1) + 2x^{\frac{1}{2}}\right)^{\frac{1}{2}} \stackrel{2.P}{=} \left(\left((x+1) - 2x^{\frac{1}{2}}\right) \cdot \left((x+1) + 2x^{\frac{1}{2}}\right)\right)^{\frac{1}{2}} \stackrel{2.P}{=} \left((x+1)^2 - \left(2x^{\frac{1}{2}}\right)^2\right)^{\frac{1}{2}}$$

$$= x^2 + 2x + 1 - 4x^{\frac{1}{2}} \stackrel{1}{=} x^2 - 2x + 1 \stackrel{2.bF}{=} x - 1 \stackrel{3.P}{=} x - 1 \quad (5)$$

$$q) a^{-b} \sqrt{x^{a^2-2ab+b^2}} : a^b \sqrt{x^{a^2+2ab+b^2}} \stackrel{D+1.+2.bF}{=} \left(x^{(a-b)^2}\right)^{\frac{1}{a-b}} : \left(x^{(a+b)^2}\right)^{\frac{1}{a+b}} \stackrel{3.P}{=} x^{a-b} : x^{a+b} \stackrel{1.P}{=} x^{a-b-(a+b)} = x^{-2b} \quad (2)$$

$$r) (\sqrt{a} + 1)(1 - \sqrt[4]{a})(1 + \sqrt[4]{a}) + a \stackrel{3.bF+3.P}{=} (\sqrt{a} + 1)(1 - \sqrt{a}) + a \stackrel{3.bF+3.P}{=} 1 - a + a = 1 \quad (2)$$

$$s) (25^{\frac{1}{3}} - 4^{\frac{1}{3}}) \cdot (5^{\frac{1}{3}} + 2^{\frac{1}{3}}) = 25^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} + 25^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} - 4^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} - 4^{\frac{1}{3}} \cdot 2^{\frac{1}{3}}$$

$$= 5^{\frac{2}{3}} \cdot 5^{\frac{1}{3}} + 25^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} - 4^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} - 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}$$

$$\stackrel{1.+2.P}{=} 5^{\frac{2}{3} + \frac{1}{3}} + (25 \cdot 2)^{\frac{1}{3}} - (4 \cdot 5)^{\frac{1}{3}} - 2^{\frac{2}{3} + \frac{1}{3}}$$

$$= 5 + 50^{\frac{1}{3}} - 20^{\frac{1}{3}} - 2$$

$$= 3 + 50^{\frac{1}{3}} - 20^{\frac{1}{3}} \quad (3)$$

Aufgabe 8: Potenzgleichungen

Bestimme die Lösungsmenge der folgenden Gleichungen:

a) $(x-5)^4 - 81 = 0$

b) $(x+3)^4 - 16 = 0$

c) $(x-2)^4 - 256 = 0$

d) $(2x-1)^4 = 16$

e) $(3x-1)^4 = 81$

f) $2 \cdot (3x-2)^3 = 4$

g) $3 \cdot (2x-3)^5 = 6$

h) $x^{\frac{3}{4}} = 2^{-3}$

i) $\sqrt[6]{x^5} = 10^{-5}$

j) $\sqrt[3]{x^2} = 2^4$

k) $\sqrt[3]{x^4} = 2^{-4}$

l) $\sqrt[3]{x-1} = 2$

Lösungen

a) $(x-5)^4 - 81 = 0 \Leftrightarrow x + 5 = \pm 3 \Rightarrow L = \{2; 8\}$ (2)

b) $(x+3)^4 - 16 = 0 \Leftrightarrow x + 3 = \pm 2 \Rightarrow L = \{-1; -5\}$ (2)

c) $(x-2)^4 - 256 = 0 \Leftrightarrow x - 2 = \pm 4 \Rightarrow L = \{-2; 6\}$ (2)

d) $(2x-1)^4 = 16 \Leftrightarrow 2x-1 = \pm 2 \Rightarrow L = \{-\frac{1}{2}; \frac{3}{2}\}$ (2)

e) $(3x-1)^4 = 81 \Leftrightarrow 3x-1 = \pm 3 \Rightarrow L = \{-\frac{2}{3}; \frac{4}{3}\}$ (2)

f) $2 \cdot (3x-2)^3 = 4 \Leftrightarrow 3x-2 = \sqrt[3]{2} \Leftrightarrow x = \frac{\sqrt[3]{2}}{3} + \frac{2}{3}$ (2)

g) $3 \cdot (2x-3)^5 = 6 \Leftrightarrow (2x-3) = \sqrt[5]{2} \Leftrightarrow x = \frac{\sqrt[5]{2}}{2} + \frac{3}{2}$ (2)

h) $x^{\frac{3}{4}} = 2^{-3} \Leftrightarrow x = 2^{-3 \cdot \frac{4}{3}} = 2^{-4} = \frac{1}{16} \Leftrightarrow L = \{\frac{1}{16}\}$ (1)

i) $\sqrt[6]{x^5} = 10^{-5} \Leftrightarrow x^{\frac{5}{6}} = 10^{-5} \Leftrightarrow L = \{10^{-6}\}$ (1)

j) $\sqrt[3]{x^2} = 2^4 \Leftrightarrow x^{\frac{2}{3}} = 2^4 \Leftrightarrow x = 2^{4 \cdot \frac{3}{2}} \Leftrightarrow L = \{64\}$ (1)

k) $\sqrt[3]{x^4} = 2^{-4} \Leftrightarrow x^{\frac{4}{3}} = 2^{-4} \Leftrightarrow x = 2^{-4 \cdot \frac{3}{4}} \Leftrightarrow L = \{\frac{1}{8}\}$ (1)

l) $\sqrt[3]{x-1} = 2 \Leftrightarrow x-1 = 2^3 \Rightarrow L = \{9\}$ (1)

Lösung von Exponentialgleichungen durch Exponentenvergleich

Berechne die Lösung der folgenden Gleichungen: $\frac{3^{2x-1}}{27} = 9^{2x+3}$.

a) $3^{x^2-2} = \frac{9}{3^{3x}}$ (3)

b) $\frac{1}{2} 2^{x^2-1} = \frac{16}{2^x}$ (3)

c) $\frac{3^{2x-1}}{27} = 9^{2x+3}$ (4)

Lösungen:

a) $3^{x^2-2} = \frac{9}{3^{3x}} \Leftrightarrow 3^{x^2-2} = 3^{2-3x}$ (1)

$\Leftrightarrow x^2 - 2 = 2 - 3x \Leftrightarrow x^2 + 3x - 4 = 0$ (1)

$\Leftrightarrow (x-1)(x+4) = 0 \Leftrightarrow x_1 = 1 \text{ und } x_2 = -4$ (1)

b) $\frac{1}{2} 2^{x^2-1} = \frac{16}{2^x} \Leftrightarrow 2^{x^2-2} = 2^{4-x}$ (1)

$\Leftrightarrow x^2 - 2 = 4 - x \Leftrightarrow x^2 + x - 6 = 0$ (1)

$\Leftrightarrow (x-2)(x+3) = 0 \Leftrightarrow x_1 = 2 \text{ und } x_2 = -3$ (1)

c) $\frac{3^{2x-1}}{27} = 9^{2x+3} \Leftrightarrow 3^{2x-1} = 9^{2x+3} \cdot 27 \Leftrightarrow 3^{2x-1} \Leftrightarrow (3^2)^{2x+3} \cdot 3^3$ (1)

$\Leftrightarrow 3^{2x-1} = 3^{2 \cdot (2x+3) + 3}$ (1)

$\Leftrightarrow 2x - 1 = 2(2x + 3) + 3$ (1)

$\Leftrightarrow 2x - 1 = 4x + 9 \Leftrightarrow -10 = 2x \Leftrightarrow x = -5$. (1)