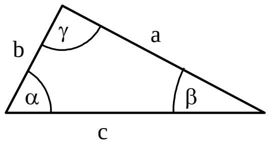
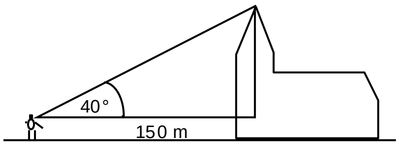
4.8. Exercises on trigonometric functions

Exercise 1: Right-angled triangles
Find the missing lengths and angles.
The lengths are given in cm.



part	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)	l)
a	4		36	4,5		2,5	8,6	5	3.9	27,2	17,3	
b			13,20		7,2			2				
c		8,61		7,6			13,2		4.6			35,2
α	48°				54°						23°	
β		64°				56°				36°		53°

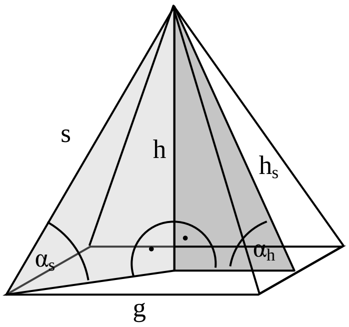
Exercise 2: Applications



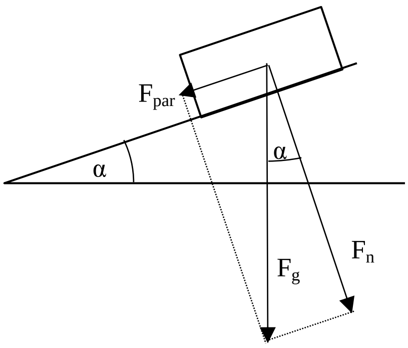
a) An observer stands 150 from the base of a church tower and sees its top at an elevation of 9°. His eyes are 1,5 m above ground. How high is the tower?
b) A ladder with length 7,5 m leans to a wall and meets it 6,6 m above the floor. How far is its base from the wall and what is its angle of inclination to the floor?
c) A radio mast is to be secured with 20 m ropes which are to have an elevation of 65°. How far above ground will they join the mast? How far from the mast will they be fastened in the ground?
d) How high is a fir, when its shadow is 27,5 m long and the sun stands at an elevation of 38,5° above the horizon?
e) How far comes a paraglider who has started from a height of 25 m at a straight flightpath with a depression of 8°?
f) From the top of a 28,6 m high tower the adjacent river appears at an angular width of 17°. Its nearest bank is 6 m from the foot of the tower. How broad is the river?
g) For an observer 12 m away a flagpole on top of a 15 m high tower appears at an angular length of 6,5°. How long is the flagpole?

Exercise 3: pyramids
Find the missing measures. All lengths are given in cm

part	a)	b)	c)	d)	e)	f)	g)	h)	i)
g	5			8				10	
s	4	6							9
h		4	4		5		5		
h_s			5			7			
α_s				70°			45°		60°
α_h					45°	60°		50°	



Exercise 4: Resolution of forces on an inclined plane
A boy with mass m = 20 kg sits on a slide with depression α = 30°. His acceleration is caused by the parallel component F_{par} of the gravitational force $F_g = m \cdot g$ with the gravitational field constant g = 9,81 m/s^2. Find the magnitude of F_{par}.



Exercise 5: Intersection angle of two lines
Find the intersection angle of each pair of lines:

a) $g_1(x) = x - 1$ und $g_2(x) = \frac{1}{2}x + 1$
b) $g_1(x) = 2x - 3$ und $g_2(x) = x$
c) $g_1(x) = -\frac{2}{3}x + 1$ und $g_2(x) = -2x + 4$
d) $g_1(x) = -x + 5$ und $g_2(x) = 3x - 2$

Exercise 6: Measuring angles in radians
Fill in the gaps:

degree	0°		45°		90°		135°		180°		360°		57,29°	70°	
radian		$\frac{\pi}{6}$		$\frac{\pi}{3}$		$\frac{2\pi}{3}$		$\frac{5\pi}{6}$		$\frac{3\pi}{2}$		$\frac{\pi}{9}$			2

Exercise 7: Transforming the sine curve
Find the angular velocity ω, the period T, the amplitude A, the phase t_0 and the vertical shift y_0 of the following functions. Draw the graphs of f_1 - f_3 in a common coordinate system with the domain $-4 \le t \le 4$

a) $f_1(t) = \sin[2\pi t]$
$f_2(t) = \sin[\pi t]$
$f_3(t) = \sin[\frac{\pi}{2} t]$

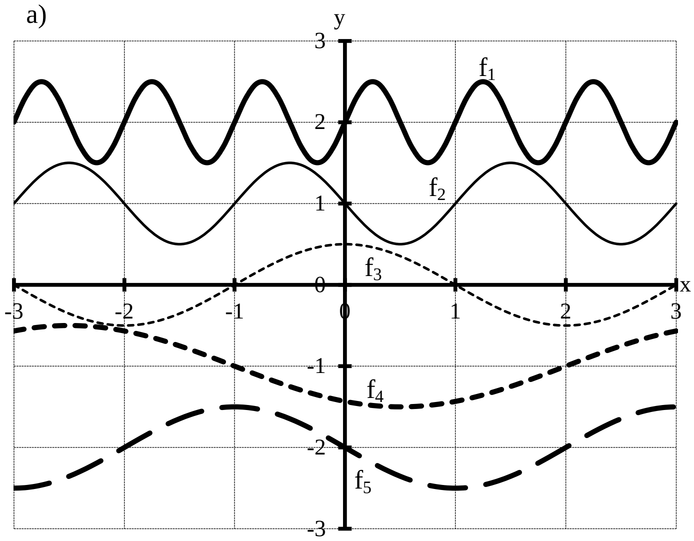
b) $f_1(t) = \frac{1}{2}\sin[2\pi t]$
$f_2(t) = 2\sin[\frac{\pi}{3} t]$
$f_3(t) = 3\sin[\frac{2\pi}{3} t]$

c) $f_1(t) = \sin[2\pi(t-1)] + 2$
$f_2(t) = \sin[\pi(t - \frac{1}{2})]$
$f_3(t) = \sin[\frac{\pi}{2}(t+1)] - 2$

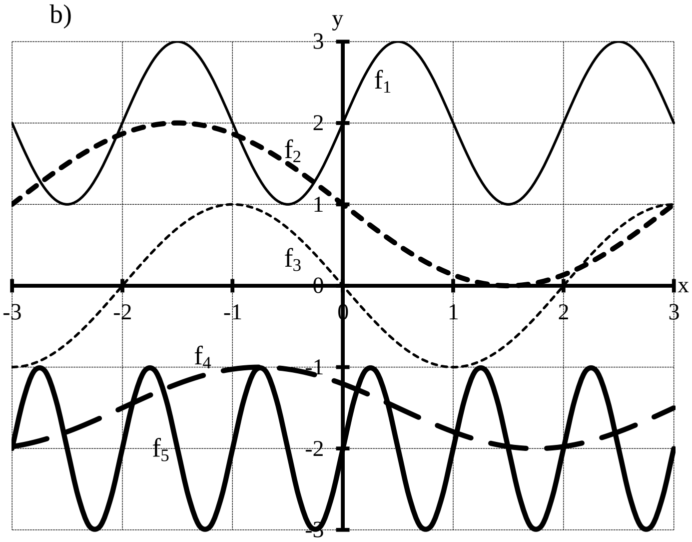
d) $f_1(t) = \frac{3}{2}\sin[\frac{\pi}{3}(t-2)] + 2$
$f_2(t) = \frac{1}{2}\sin[\frac{2\pi}{3}(t-1)]$
$f_3(t) = \frac{3}{2}\sin[\pi(t+1)] - 2$

Exercise 8: Transforming the sine curve
Find the formulae of the functions f_1 - f_5:

a)



b)



Exercise 9: Sine rule
Find the missing data of a triangle with sides a, b, c and angles α, β, γ with values given as follows:
a) a = 14,3 m; c = 27,9 m und γ = 82,1°
b) a = 13 m; b = 27 m und α = 27°

Exercise 10: Sine rule
Show that the bisector in a triangle divides the opposite side in the ratio of the adjacent sides.

Exercise 11: Sine and Cosine rule
Find the missing measures of a triangle with sides a, b, c and angles α, β, γ with values given as follows:
a) α = 30°,β = 60°, a = 3 cm
b) a = 6 cm, b = 4 cm, γ = 40°
c) a = 3 cm; b = 4 cm; c = 5 cm

Exercise 12: Cosine rule
To avoid an inaccessible rock, a land surveyor starts at point A, walks 85 m due north to point B and continues in a straight line for 102 m on a bearing of 052° until point C. Find the distance between A and C.

4.8. Solutions tot he exercises on trigonometric functions

Exercise 1: Right-angled triangles

part	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)	l)
a	4	3,77	36	4,5	9,91	2,5	8,6	5	3,9	27,2	17,3	21,18
b	3,6	7,74	13,2	6,12	7,2	3,71	10,01	2	2,44	19,76	40,75	28,11
c	5,38	8,61	38,34	7,6	12,25	4,47	13,20	5,38	4,6	33,62	44,28	35,2
α	48°	26°	69,86	36,31°	54°	34°	40,66	68,20	57,98°	54°	23°	37°
β	42°	64°	20,13	53,69°	36°	56°	49,34	21,80	32,02°	36°	67°	53°

Worked example to a):

$\beta = 90° - 48° = 42°$

$c = \frac{a}{\sin(\alpha)} = \frac{4\text{cm}}{0,74} \approx 5,38$ cm

$b = c\cdot\cos(\beta) = 5,38\text{ cm}\cdot 0,67 \approx 3,6$ cm

Exercise 2: Applications

a) Height $h = 1,5\text{ m} + 150\text{ m}\cdot\tan(40°) = 126,5$ m.

b) Inclination angle $\alpha = \sin^{-1}(\frac{6,6}{7,5}) \approx 61,64°$ and distance $d \approx \sqrt{7,5^2 - 6,6^2} \approx 3,56$ m

c) Height $h = 20\text{ m}\cdot\sin(65°) \approx 18,12$ m und distance $d = 20\text{ m}\cdot\cos(65°) \approx 8,45$ m

d) Height $h = 27,5\text{ m}\cdot\tan(38,5°) \approx 21,87$ m

e) distance $d = \frac{25\text{ m}}{\tan(8°)} \approx 1777,88$ m

f) The nearest shore appears at the angle $\alpha_1 = \tan^{-1}(\frac{6\text{ m}}{28,6\text{ m}}) \approx 11,84°$ to the vertical and has the distance $d_1 = 6$ m from the tower. The farther shore appears at the angle $\alpha_2 = 17° + 11,84° = 28.84°$ to the vertical and has the distance $d_2 = 28,6\text{ m}\cdot\tan(28,84°) \approx 15,75$ m from the tower. So the river has a width of $d_2 - d_1 = 9,75$ m.

g) The lower end of the flagpole appears at the angle $\alpha_1 = \tan^{-1}(\frac{13,4\text{ m}}{12\text{ m}}) \approx 48,15°$ to the horizontal and is $h_1 = 13,4$ m above the eyes of the observer. The upper end appears at the angle $\alpha_2 = 6,5° + 48,15° = 54,65°$ und is $h_2 = 12\text{ m}\cdot\tan(54,65°) \approx 16,92$ m above the eyes of the observer. Therefore the flagpole has a height of $h_2 - h_1 = 3,52$ m.

Exercise 3: pyramids

part	a)	b)	c)	d)	e)	f)	g)	h)	i)
g	5	6,32	6	8	10	7	7,07	10	6,36
s	4	6	5,83	16,53	8,66	7,83	7,07	9,25	9
h	1,87	4	4	15,54	5	6,06	5	5,96	7,79
h_s	3,12	5,10	5	16,05	7,07	7	6,12	7,78	8,42
α_s	27,87°	41,81°	43,32°	70°	35,37°	50,71°	45°	40,12°	60°
α_h	36,82°	51,66°	53,13°	75,52°	45°	60°	54,78°	50°	67,70°

Worked example to a)

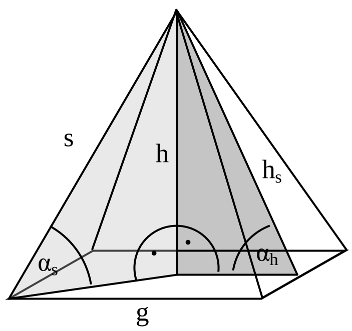
The base quadrangle has the diagonal $d = \sqrt{g^2 + g^2} = \sqrt{2}\,g = \sqrt{2}\cdot 4 \approx 5,65$ cm

Height $h = \sqrt{s^2 - \left(\frac{d}{2}\right)^2} \approx \sqrt{4^2 - \left(\frac{5,65}{2}\right)^2} \approx 1,87$ cm (light shaded triangle)

Face height $h_s = \sqrt{h^2 + \left(\frac{g}{2}\right)^2} \approx \sqrt{1,87^2 + \left(\frac{5}{2}\right)^2} \approx 3,12$ cm (dark shaded trangle)

Corner angle $\alpha_s = \sin^{-1}\left(\frac{h}{s}\right) \approx \sin^{-1}\left(\frac{1,87\text{ cm}}{4\text{ cm}}\right) \approx 27,87°$ (light shaded triangle)

Edge angle $\alpha_h = \sin^{-1}\left(\frac{h}{h_s}\right) \approx \sin^{-1}\left(\frac{1,87\text{ cm}}{3,12\text{ cm}}\right) \approx 36,82°$ (dark shaded trangle)



Exercise 4: Resolution of forces on an inclined plane

$F_h = F_g \cdot \sin(\alpha) = mg \cdot \sin(\alpha) = 98{,}1$ N (corresponds to the gravitational pull of 10 kg)

Exercise 5: Intersection angles of lines

a) $\alpha = 45° - 26{,}5° = 18{,}5°$

b) $\alpha = 63{,}43° - 45° = 18{,}43°$

c) $\alpha = -33{,}69° - (-63{,}43°) = 29{,}74°$

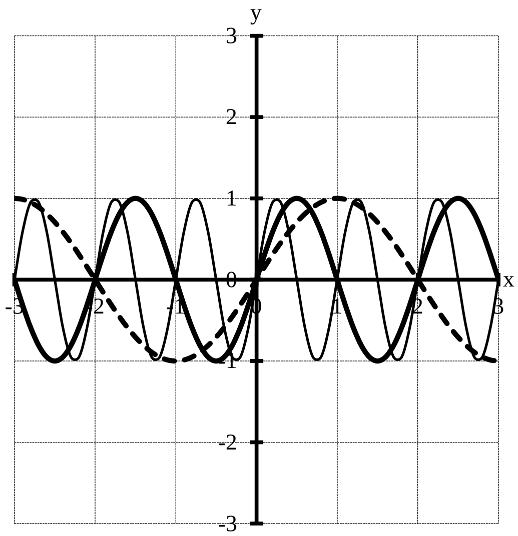
d) $\alpha = 71{,}57° - (-45°) = 116{,}57°$ bzw $63{,}43°$

Exercise 6: Measuring angles in degrees and radians

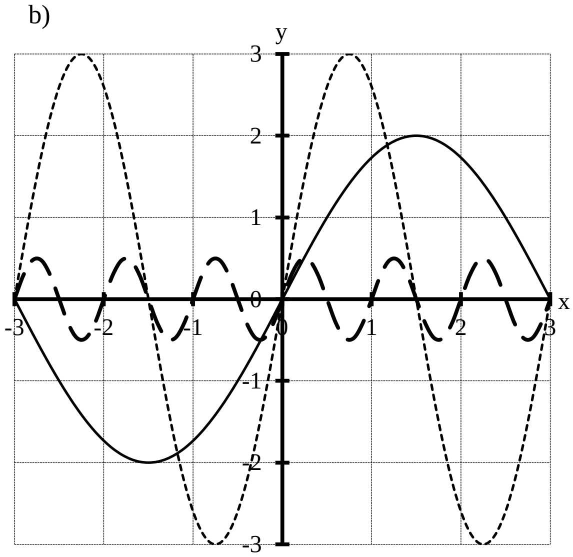
degree	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°	20°	57,29°	70°	114,59°
radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π	$\frac{\pi}{9}$	1	$\frac{7\pi}{18}$	2

Exercise 7: Transforming the graph of the sine function

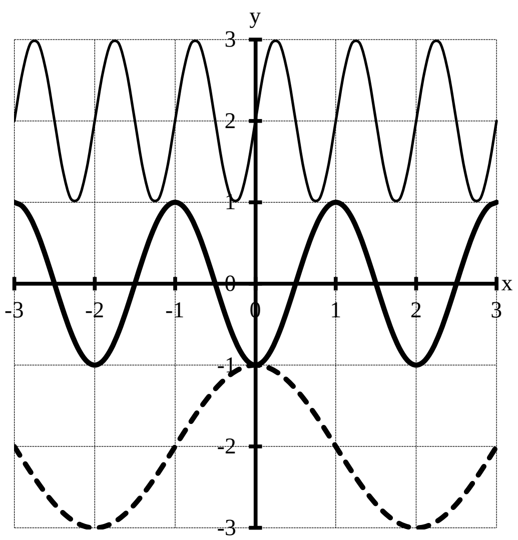
a)



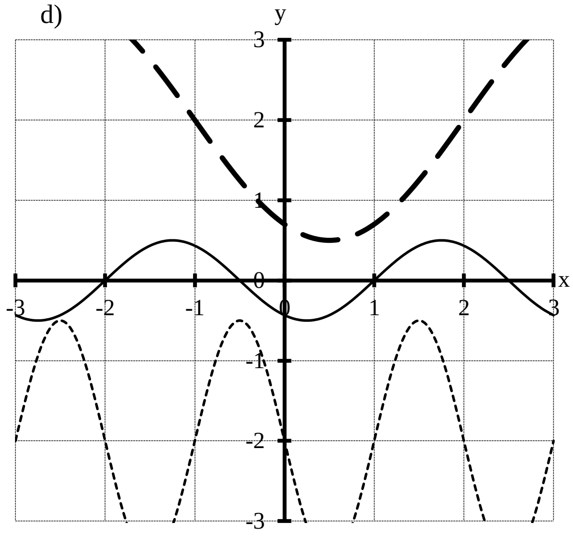
b)



c)



d)



Exercise 8: Transforming the graph of the sine function

a)

$f_1(t) = \frac{1}{2}\sin[2\pi t] + 2$

$f_2(t) = \frac{1}{2}\sin[\pi\,(t - 1)] + 1$

$f_3(t) = \frac{1}{2}\sin[\frac{\pi}{2}\,(t + 1]$

$f_4(t) = \frac{1}{2}\sin[\frac{\pi}{3}\,(t + 2)] - 1$

$f_5(t) = \frac{1}{2}\sin[\frac{\pi}{2}\,(t - 2)] - 2$

b)

$f_1(t) = \sin[\pi x] + 2$

$f_2(t) = \sin[\frac{\pi}{3}\,(t + 3)] + 1$

$f_3(t) = \sin[\frac{\pi}{2}\,(t + 2]$

$f_4(t) = \sin[\frac{2\pi}{5}\,(t + 2)] - \frac{3}{2}$

$f_5(t) = \sin[2\pi t] - 2$

Exercise 9: Sine rule

a) $\sin(\alpha) = a\cdot\frac{\sin(\gamma)}{c} = 0{,}51 \Rightarrow \alpha = 30{,}5°$ (Sine rule)

$\beta = 180° - \alpha - \gamma = 67{,}39°$ (angle sum)

$b = a\cdot\frac{\sin(\beta)}{\sin(\alpha)} = 26{,}00$ m (Sine rule)

b) $\sin(\beta) = b\cdot\frac{\sin(\alpha)}{a} = 0{,}94 \Rightarrow \beta = 70{,}54°$ (Sine rule)

$\gamma = 180° - \alpha - \beta = 82{,}46°$ (angle sum)

$c = a\cdot\frac{\sin(\gamma)}{\sin(\alpha)} = 28{,}38$ m (Sine rule)

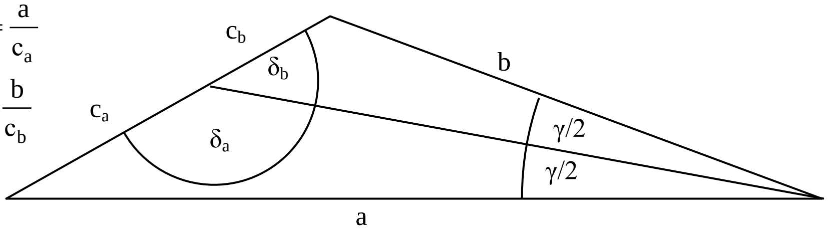
Exercise 10: Sine rule

In the lower triangle we have $\frac{\sin(\delta_a)}{\sin(\gamma/2)} = \frac{a}{c_a}$

In the upper trangle we have $\frac{\sin(\delta_b)}{\sin(\gamma/2)} = \frac{b}{c_b}$

Because of $\delta_a + \delta_b = 180°$ we can write $\sin(\delta_a) = \sin(\delta_b)$ so that

$\frac{b}{c_b} = \frac{a}{c_a} \Leftrightarrow \frac{c_a}{c_b} = \frac{a}{b}$, qed.



Exercise 11: Sine and Cosine rules

a) $\gamma = 180° - \alpha - \beta = 90°$ (angle sum)

$b = a\cdot\frac{\sin(\alpha)}{\sin(\beta)} = 5{,}2$ cm (Sine rule)

$c = a\cdot\frac{\sin(\gamma)}{\sin(\alpha)} = 6$ cm (Sine rule)

b) $c = \sqrt{a^2 + b^2 - 2a\cdot b\cdot\cos(\gamma)} = 3{,}9$ cm (Cosine rule)

$\sin(\beta) = b\cdot\frac{\sin(\gamma)}{c} = 0{,}66 \Rightarrow \beta = 41{,}24°$ (Sine rule)

$\beta = 180° - \beta - \gamma = 98{,}76°$ (angle sum)

Beware the **obtuse** angle α: $\sin(\alpha) = a\cdot\frac{\sin(\gamma)}{c} = 0{,}99$.

The calculator gives the acute neighbor angle $\sin^{-1}(0.99) = 180° - \alpha = 81{,}36°$!

c) $\cos(\alpha) = \frac{a^2 - b^2 - c^2}{2bc} = \frac{4}{5} \Rightarrow \alpha = 36{,}8°$ (Cosine rule)

$\cos(\beta) = \frac{b^2 - a^2 - c^2}{2ac} = \frac{3}{5} \Rightarrow \beta = 53{,}1°$ (Cosine rule)

$\gamma = 180° - \alpha - \beta = 90°$ (angle sum)

Exercise 12: Cosine rule

$\beta = 180° - 52{,}2° = 127{,}8°$ (neighbor angle)

$\overline{AC} = \sqrt{\overline{AB}^2 - \overline{BC}^2 - 2\overline{AB}\cdot\overline{BC}\cdot\cos(\beta)} = 167{,}88$ m. (Cosine rule)