7.2. Exercises on lines in space

Exercise 1: Change of support and direction vectors

1. Check whether P or Q are on g.
2. Use the point found in 1. to find both a new support vector and a new direction vector for g.
3. Assuming that g describes the motion of a particle, compare the speeds with the old and the new direction vector.

a) P(−1|1|1), Q(−1|4|3) and g: $\vec{x} = \begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}-1\\2\\1\end{pmatrix}$ b) P(2|0|−1), Q(−1|2|0) and g: $\vec{x} = \begin{pmatrix}1\\-1\\0\end{pmatrix} + t\begin{pmatrix}1\\1\\-1\end{pmatrix}$

Exercise 2: Intersects with the coordinate planes

Find the intersects with the three coordinate planes E_{12}: $x_3 = 0$; E_{13}: $x_2 = 0$ and E_{23}: $x_1 = 0$. Use the intersects to draw a sketch of the line in a three dimensional coordinate system.

a) g: $\vec{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + r\begin{pmatrix}-1\\2\\1\end{pmatrix}$ b) g: $\vec{x} = \begin{pmatrix}1\\3\\-1\end{pmatrix} + s\begin{pmatrix}-1\\-2\\1\end{pmatrix}$ c) g: $\vec{x} = \begin{pmatrix}1\\-2\\3\end{pmatrix} + r\begin{pmatrix}1\\1\\2\end{pmatrix}$ d) g: $\vec{x} = \begin{pmatrix}-2\\3\\5\end{pmatrix} + s\begin{pmatrix}1\\1\\0\end{pmatrix}$

Exercise 3: Line with given direction through a given point

Find the line through the point P in the direction of $\vec{b}$. Determine the speed of the particle.

a) P(1|0|1) and $\vec{b} = \begin{pmatrix}-1\\2\\0\end{pmatrix}$ b) P(−5|6|7) and $\vec{b} = \begin{pmatrix}-7\\5\\9\end{pmatrix}$ c) P(2|1|0) and $\vec{b} = -\vec{i} - \vec{j} + 2\vec{k}$ d) P(1|0|0) and $\vec{b} = \vec{k}$

Exercise 4: Line with given direction through a given point

a) Find the vector equation of the line h through the point A(2|1|4) parallel to $\vec{b} = 2\vec{i} - 3\vec{j} + 6\vec{k}$.
b) Calculate the magnitude of the direction vector.
c) Find the coordinates of the point P on h with AP = 42.

Exercise 5: Line through a two given points

Find the vector equation of the line h through the points P and Q:

a) P(−1|1|1) and Q(1|4|3) b) P(1|0|0) and Q(0|1|0) c) P(−1|5|0) and Q(2|0|−1)

Exercise 6: Line through a two given points

a) Find the vector equation of the line h through the points P(7|1|2) and Q(3|−1|5).
b) Find the coordinates of the point R on h with PR = 3·PQ.

Exercise 7: Relations between lines

1. Find the intersects with the three coordinate planes E_{12}: $x_3 = 0$; E_{13}: $x_2 = 0$ and E_{23}: $x_1 = 0$.
2. Determine if the two lines g and h intersect and if they do, find the intersection point.
3. Find the angle between the two lines and decide whether they are parallel, perpendicular, skew or none of the above.

a) g: $\vec{x} = \begin{pmatrix}1\\0\\1\end{pmatrix} + r\begin{pmatrix}-1\\2\\0\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}1\\-1\\0\end{pmatrix} + s\begin{pmatrix}1\\1\\-1\end{pmatrix}$ b) g: $\vec{x} = \begin{pmatrix}1\\2\\3\end{pmatrix} + r\begin{pmatrix}1\\-1\\0\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}2\\0\\1\end{pmatrix} + s\begin{pmatrix}-1\\3\\4\end{pmatrix}$

c) g: $\vec{x} = \begin{pmatrix}1\\0\\1\end{pmatrix} + r\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}1\\-3\\1\end{pmatrix} + s\begin{pmatrix}2\\4\\0\end{pmatrix}$ d) g: $\vec{x} = \begin{pmatrix}-2\\3\\5\end{pmatrix} + r\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}0\\1\\9\end{pmatrix} + s\begin{pmatrix}-1\\2\\-4\end{pmatrix}$

e) g: $\vec{x} = \begin{pmatrix}0\\0\\3\end{pmatrix} + r\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}2\\1\\0\end{pmatrix} + s\begin{pmatrix}2\\1\\-1\end{pmatrix}$ f) g: $\vec{x} = \begin{pmatrix}1\\1\\2\end{pmatrix} + r\begin{pmatrix}-1\\2\\2\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}1\\2\\4\end{pmatrix} + s\begin{pmatrix}2\\-4\\-4\end{pmatrix}$

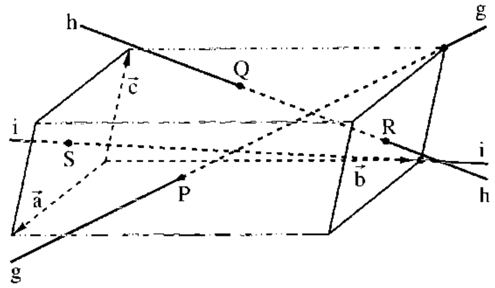
g) g: $\vec{x} = \begin{pmatrix}3\\6\\2\end{pmatrix} + r\begin{pmatrix}2\\3\\1\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}-1\\-4\\-1\end{pmatrix} + s\begin{pmatrix}2\\-1\\0\end{pmatrix}$ h) g: $\vec{x} = \begin{pmatrix}2\\2\\1\end{pmatrix} + r\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and h: $\vec{x} = \begin{pmatrix}-2\\6\\5\end{pmatrix} + s\begin{pmatrix}2\\-2\\4\end{pmatrix}$

Exercise 8: Relations between lines
Find all intersection points of the three lines, g, h and l:

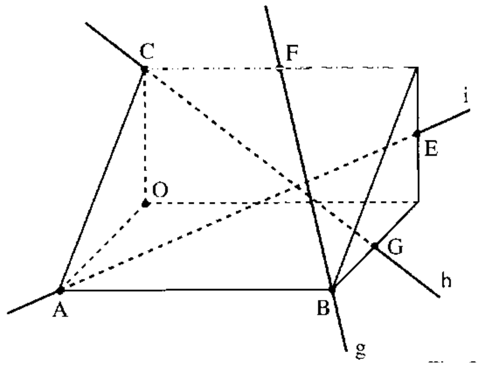
a) g: $\vec{x} = \vec{i} + t(\vec{j} - \vec{i})$, h: $\vec{x} = \vec{i} + \vec{j} + t(\vec{i} - \frac{1}{2}\vec{j})$ and l: $\vec{x} = \frac{1}{2}\vec{i} + \vec{j} + t(\vec{i} - \vec{j})$.

b) g: $\vec{x} = \vec{i} + \vec{j} + t(\vec{i} + \vec{j} + \vec{k})$, h: $\vec{x} = \vec{j} + t(\vec{i} - \frac{1}{2}\vec{j} - \frac{1}{2}\vec{k})$ and l: $\vec{x} = \vec{i} - \vec{k} + t(\vec{j} + \vec{k})$.

Exercise 9: Relations between lines in a parallelepiped
The points P, Q, R and S are centroids of the side surfaces in the parallelepiped on the right. Determine the relation of the lines g, h and i and find all intersection points.



Exercise 10: Relations between lines in a prism
The points E, F and G are centres of the edges in the parallelepiped on the right. Determine the relation of the lines g, h and i and find all intersection points.



Exercise 11: Motion on a straight line
In this question the unit vectors $\vec{i}$ and $\vec{j}$ point due East and North, respectively.
A port is located at the origin. One ship starts from the port and moves with velocity $\vec{v}_1 = (3\vec{i} + 4\vec{j})$ km·h^{-1}.

a) Write down the position vector at time t hours.
b) At the same time, a second ship starts 18 km north of the port and moves with velocity $\vec{v}_2 = (3\vec{i} - 5\vec{j})$ km·h^{-1}.Write down the position vector of the second ship at time t hours.
c) Show that after half an hour the distance between the two ships is 13,5 km.
d) Show that the ships meet and find the time when this happens.
e) How long after the meeting are the ships 18 km apart?

Exercise 12: Motion on a straight line
At time t = 0, two aircraft have position vectors $5\vec{j}$ and $7\vec{k}$. The first moves with velocity $3\vec{i} - 4\vec{j} + \vec{k}$ and the second with velocity $5\vec{i} + 2\vec{j} - \vec{k}$.

a) Write down the position vector of the first aircraft at time t.
b) Show that at time t the distance, d, between the two aircraft is given by $d^2 = 44t^2 - 88t + 74$.
c) Show that the two aircraft will not collide.
d) Find the minimum distance between the two aircraft.

Exercise 13: Cartesian equation of a line
a) Write down the Cartesian equation of the lines g and h in Exercise 7 a) and b).
b) Write down the vector equation of the lines g: $\frac{x_1 - 3}{2} = \frac{x_2 + 1}{-4} = \frac{x_3}{5}$ and h: $\frac{x_1 + 1}{5} = \frac{3 - x_3}{2}$, $x_2 = 1$.

Exercise 14: Cartesian equation of a line
1. Determine whether the following pairs of lines are parallel, perpendicular, the same line or none of the above.
2. Find the intersection points of each pair.

a) $\vec{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ and $\vec{r} = 4\vec{i} - \vec{k} + \mu(-2\vec{i} + \vec{j} + 3\vec{k})$

b) $\vec{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix}$ and $\frac{2x_1 - 1}{4} = \frac{x_2 - 2}{-3} = \frac{6 - x_3}{2}$

c) $\frac{x_1 - 5}{7} = \frac{x_2 + 1}{-1} = 4 - x_3$ and $x_1 = -3\lambda + 1$, $x_2 = 0$, $x_3 = 6 - \lambda$

d) $x_1 = 2t + 1$, $x_2 = 1 - 4t$, $x_3 = 3$ and $\vec{r} = \begin{pmatrix} 8 \\ -13 \\ 3 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

e) $\frac{x_1 - 2}{3} = \frac{x_2 + 1}{4} = x_3 + 1$ and $5 - x_1 = \frac{x_2 + 2}{-3} = \frac{x_3 - 7}{2}$

e) $\vec{r} = \begin{pmatrix} 7 \\ 8 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\frac{x_1 - 2}{3} = \frac{x_2 + 1}{4} = x_3 + 1$

7.2. Solutions to the exercises on lines in space

Exercise 1: Change of support and direction vectors

a) P ∉ g because the equation $\overrightarrow{PQ} = \vec{x} \Leftrightarrow \begin{pmatrix}-1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} + r\begin{pmatrix}-1\\2\\1\end{pmatrix} \Leftrightarrow \begin{pmatrix}-2\\1\\0\end{pmatrix} = r\begin{pmatrix}-1\\2\\1\end{pmatrix}$ has no solution

Q ∈ g because the equation $\overrightarrow{OQ} = \vec{x}$ has the solution r = 2 (Q is reached after r = 2 time units).

With new supporting vector $\overrightarrow{OQ} = \begin{pmatrix}-1\\4\\3\end{pmatrix}$ and new direction vector $\begin{pmatrix}-1\\4\\3\end{pmatrix} - \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-2\\4\\2\end{pmatrix}$ we have g: $\vec{x} = \begin{pmatrix}-1\\4\\3\end{pmatrix} + r\begin{pmatrix}-2\\4\\2\end{pmatrix}$

with double speed $\sqrt{2^2+4^2+2^2} = 2\sqrt{6}$

b) P ∈ g because the equation $\overrightarrow{OP} = \vec{x}$ has solution r = 1. (Q is reached after r = 1 time units).

Q ∉ g because the equation $\overrightarrow{OQ} = \vec{x} \Leftrightarrow \begin{pmatrix}-1\\2\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix} + r\begin{pmatrix}1\\1\\-1\end{pmatrix} \Leftrightarrow \begin{pmatrix}-2\\-3\\0\end{pmatrix} = r\begin{pmatrix}1\\1\\-1\end{pmatrix}$ has no solution

With new supporting vector $\overrightarrow{OP} = \begin{pmatrix}2\\0\\-1\end{pmatrix}$ and e.g. reversed direction vector $\begin{pmatrix}-1\\-1\\1\end{pmatrix}$ we have g: $\vec{x} = \begin{pmatrix}2\\0\\-1\end{pmatrix} + r\begin{pmatrix}1\\1\\-1\end{pmatrix}$ with same speed $\sqrt{3}$. Because Q is reached after 1 time unit, the vector $\begin{pmatrix}2\\0\\-1\end{pmatrix} - \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\-1\end{pmatrix}$ from starting point to Q is again the original velocity.

Exercise 2: Intersects with the coordinate planes

a) $S_{12}(2|-1|0)$ for r = −1, $S_{13}(\frac{3}{2}|0|\frac{1}{2})$ for $r = -\frac{1}{2}$ and $S_{23}(0|3|2)$ for r = 1

b) $S_{12}(0|1|0)$ for r = 1, $S_{13}(-\frac{1}{2}|0|\frac{1}{2})$ for $r = \frac{3}{2}$ and $S_{23}(0|1|0)$ for r = 1

c) $S_{12}(-\frac{1}{2}|-\frac{7}{2}|0)$ for $r = -\frac{3}{2}$, $S_{13}(3|0|7)$ for r = 2 and $S_{23}(0|-3|1)$ for r = −1

d) S_{12} does not exist, $S_{13}(-5|0|5)$ for r = −3 and $S_{23}(0|5|5)$ for r = 2

Exercise 3: Line with given direction through a given point

a) g: $\vec{x} = \begin{pmatrix}1\\0\\1\end{pmatrix} + r\begin{pmatrix}-1\\2\\0\end{pmatrix}$ b) g: $\vec{x} = \begin{pmatrix}-5\\6\\7\end{pmatrix} + r\begin{pmatrix}-7\\5\\9\end{pmatrix}$ c) g: $\vec{x} = \begin{pmatrix}2\\1\\0\end{pmatrix} + r\begin{pmatrix}-1\\-5\\2\end{pmatrix}$ d) g: $\vec{x} = \begin{pmatrix}1\\0\\0\end{pmatrix} + r\begin{pmatrix}0\\0\\1\end{pmatrix}$

Exercise 4: Line with given direction through a given point

a) g: $\vec{x} = \begin{pmatrix}2\\1\\4\end{pmatrix} + r\begin{pmatrix}2\\-3\\6\end{pmatrix}$ b) speed is 7 c) Since 42 = 6·7 we need 6 time units to reach P ⇒ $\overrightarrow{OP} = \begin{pmatrix}2\\1\\4\end{pmatrix} + 6\begin{pmatrix}2\\-3\\6\end{pmatrix} = \begin{pmatrix}14\\-17\\40\end{pmatrix}$.

Exercise 5: Line through two given points

a) g: $\vec{x} = \begin{pmatrix}-1\\1\\1\end{pmatrix} + r\begin{pmatrix}2\\3\\2\end{pmatrix}$ b) g: $\vec{x} = \begin{pmatrix}1\\0\\0\end{pmatrix} + r\begin{pmatrix}-1\\1\\0\end{pmatrix}$ c) g: $\vec{x} = \begin{pmatrix}-1\\5\\0\end{pmatrix} + r\begin{pmatrix}3\\-5\\-1\end{pmatrix}$

Exercise 6: Line through two given points

a) h: $\vec{x} = \begin{pmatrix}7\\1\\2\end{pmatrix} + r\begin{pmatrix}-4\\-2\\3\end{pmatrix}$ b) $\overrightarrow{OR} = \begin{pmatrix}7\\1\\2\end{pmatrix} + 3\begin{pmatrix}-4\\-2\\3\end{pmatrix} = \begin{pmatrix}-5\\-5\\11\end{pmatrix}$.

Exercise 7: Relation between lines

a) g with $S_{23}(0|2|1)$, $S_{13}(1|0|1)$ and h with $S_{23}(0|-2|1)$, $S_{13}(2|0|-1)$, $S_{12}(1|-1|0)$ are skew with $\alpha = \arccos\left(\frac{1}{\sqrt{15}}\right) \approx 75{,}04°$

b) g with $S_{23}(0|3|3)$, $S_{13}(3|0|3)$ and h with $S_{23}(0|6|9)$, $S_{13}(2|0|1)$, $S_{12}(\frac{9}{4}|-\frac{3}{4}|0)$ intersect in $S(\frac{3}{2}|\frac{3}{2}|3)$ with r = s = 0,5

and $\alpha = \arccos\left(\frac{-4}{\sqrt{52}}\right) \approx 56{,}31°$

c) g with $S_{23}(0|-2|1)$, $S_{13}(1|0|1)$ and h with $S_{23}(0|-5|1)$, $S_{13}(\frac{5}{2}|0|1)$ are parallel.

d) g with $S_{23}(0|1|9)$, $S_{13}(-1|0|11)$, $S_{12}(-\frac{9}{2}|\frac{11}{2}|0)$ and h with $S_{23}(0|1|9)$, $S_{13}(\frac{1}{2}|0|11)$, $S_{12}(-\frac{9}{4}|\frac{11}{2}|0)$ intersect in $S(0|1|9)$

with r = 2 resp s = 0 and $\alpha = \arccos\left(\frac{-11}{\sqrt{126}}\right) \approx 11{,}49°$

e) g with $S_{23}(0|0|3) = S_{13}$, $S_{12}(-2|1|0)$ and h with $S_{23}(0|0|1) = S_{13}$, $S_{12}(2|1|0)$ are perpendicular and skew with $\alpha = \cos^{-1}(0) = 90°$

f) g with $S_{23}(0|3|4)$, $S_{13}(\frac{3}{2}|0|1)$, $S_{12}(2|1|0)$ and h with $S_{23}(0|4|6)$, $S_{13}(2|0|2)$, $S_{12}(3|-2|0)$ are parallel.

g) g with $S_{23}(0|\frac{3}{2}|\frac{1}{2})$, $S_{13}(-1|0|0) = S_{12}$ and h with $S_{23}(0|-\frac{7}{2}|-1)$, $S_{13}(-9|0|-1)$ and no S_{12} intersect in $S(-3|-3|-1)$ with r = −3

and s = −1 and $\alpha = \arccos\left(\frac{1}{\sqrt{7}}\right) \approx 67{,}79°$

h) g with $S_{23}(0|3|5)$, $S_{13}(4|0|-3)$, $S_{12}(\frac{5}{2}|\frac{3}{2}|0)$ and h with $S_{23}(0|4|1)$, $S_{13}(4|0|17)$, $S_{12}(-\frac{13}{2}|\frac{17}{2}|0)$ are identical. (set r = 2)

Exercise 8: Relations between lines

a) $g \cap h = \{S_{gh}\}$ with position vector $\overrightarrow{OS_{gh}} = -\vec{i} + 2\vec{j}$, $g \parallel i$ and $h \cap i = \{S_{gh}\}$ with position vector $\overrightarrow{OS_{hi}} = \frac{3}{2}\vec{j}$

b) $g \cap h = \{S_{gh}\}$ with $\overrightarrow{OS_{gh}} = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$, $g \cap i = \{S_{gi}\}$ with $\overrightarrow{OS_{gi}} = \vec{i} + \vec{j}$, $h \cap i = \{S_{hi}\}$ with $\overrightarrow{OS_{hi}} = \vec{i} + \frac{1}{2}\vec{j} - \frac{1}{2}\vec{k}$

Exercise 9: relations between lines in parallelepiped

g: $\vec{x} = \vec{j} + \vec{k} + r(2\vec{i} - \vec{j} - \vec{k})$, h: $\vec{x} = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + \vec{k} + s(\vec{j} - \vec{k})$ and i: $\vec{x} = \vec{j} + t(\vec{i} - 2\vec{j} + \vec{k})$

$g \cap h = \{S_{gh}\}$ with $\overrightarrow{OS_{gh}} = \frac{1}{2}\vec{i} + \frac{3}{4}\vec{j} + \frac{3}{4}\vec{k}$, i and g are skew and so are i and h.

Exercise 10: relations between lines in prism

g: $\vec{x} = \vec{i} + r(4\vec{i} - 2\vec{j} - \vec{k})$, h: $\vec{x} = \vec{j} + s(\vec{i} + \vec{j} - 2\vec{k})$ and i: $\vec{x} = \vec{k} + t(\vec{i} - 2\vec{j} + 2\vec{k})$

$g \cap i = \{S_{gi}\}$ with $\overrightarrow{OS_{gi}} = -\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$, h and g are skew and so are h and i.

Exercise 11: Motion on a straight line

a) first ship s_1: $\vec{x} = t(3\vec{i} + 4\vec{j}) = t\begin{pmatrix}3\\4\end{pmatrix}$

b) second ship: s_2: $\vec{x} = 18\vec{j} + t(3\vec{i} - 5\vec{j}) = \begin{pmatrix}0\\18\end{pmatrix} + t\begin{pmatrix}3\\-5\end{pmatrix}$

c) At time $t = \frac{1}{2}$ the first ship is in $\begin{pmatrix}1{,}5\\2\end{pmatrix}$ and the second ship in $\begin{pmatrix}1{,}5\\15{,}5\end{pmatrix}$ with distance $d = \sqrt{(1{,}5-1{,}5)^2 + (15{,}5-2)^2} = 13{,}5$.

d) $s_1 \cap s_2 = \{S_{12}\}$ with $\overrightarrow{OS_{12}} = \begin{pmatrix}6\\8\end{pmatrix}$ at t = 2

e) $d = 18 = \sqrt{(3t-3t)^2 + (18-5t-4t)^2} = 9t \Rightarrow t = 2$.

Exercise 12: Motion on a straight line

a) first aircraft: a_1: $\vec{x} = \begin{pmatrix}0\\5\\0\end{pmatrix} + t\cdot\begin{pmatrix}3\\-4\\1\end{pmatrix}$ and second aircraft: a_2: $\vec{x} = \begin{pmatrix}0\\0\\7\end{pmatrix} + t\cdot\begin{pmatrix}5\\2\\-1\end{pmatrix}$

b) distance d(t) = $\sqrt{(2t)^2+(6t-5)^2+(7-2t)^2} = \sqrt{4t^2+(36t^2-60t+25)+(49-28t+4t^2)} = \sqrt{44t^2-88t+74}$.

c) The equation $\begin{pmatrix}0\\5\\0\end{pmatrix} + t\cdot\begin{pmatrix}3\\-4\\1\end{pmatrix} = \begin{pmatrix}0\\0\\7\end{pmatrix} + t\cdot\begin{pmatrix}5\\2\\-1\end{pmatrix} \Leftrightarrow \begin{pmatrix}0\\5\\-7\end{pmatrix} = t\cdot\begin{pmatrix}2\\6\\-2\end{pmatrix} \Leftrightarrow \left|\begin{matrix}t=0\\t=5/6\\t=7/2\end{matrix}\right|$ has no solution.

d) d'(t) = $\frac{88t-88}{2\sqrt{44t^2-88t+74}}$ = 0 at t = 1 with sign change from – to + ⇒ rel Minimum at t = 1 with d(1) = $\sqrt{30}$ is also an absolute Minimum since d(0) = $\sqrt{74}$ and d → ∞ for t → ∞.

Exercise 13: Cartesian equation of a line

a) 7a) g: r = 1 – x_1 = $\frac{x_2}{2}$; x_3 = 1 and h: s = x_1 – 1 = x_2 + 1 = $-x_3$

7b): g: r = x_1 – 1 = 2 – x_2; x_3 = 3 and h: s = 2 – x_1 = $\frac{x_2}{3} = \frac{x_3-1}{4}$

b) g: $\vec{x} = \begin{pmatrix}3\\-1\\0\end{pmatrix} + t\cdot\begin{pmatrix}2\\-4\\5\end{pmatrix}$ and h: = $\begin{pmatrix}-1\\1\\3\end{pmatrix} + t\cdot\begin{pmatrix}5\\0\\-2\end{pmatrix}$

Exercise 14: Cartesian equation of a line

a) Intersection point S(0|2|5) at λ = 1 and μ = 2

b) skew and perpendicular

c) Intersection point S(−2|0|5) at μ = −1 and λ = 1

d) parallel

d) Intersection point S(8|7|1) at μ = −2 and λ = −3

e) Intersection point S(8|7|1) at μ = 1 and λ = −2